FURTHER MECHANICS / CIRCULAR MOTION & OSCILLATIONS

Motion in a Horizontal Circle

Questions by topic 15 questions · 146 marks

EXAM BOARDS Edexcel FM2 9FM0/4C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Major Y421 CIE 9231 Paper 3

Instructions

Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question gives a value. Where a question leaves g unspecified and a final answer depends on it, the solutions also give that answer for $g = 10\,\mathrm{m\,s^{-2}}$.

CIE (9231 Paper 3): papers take $g = 10\,\mathrm{m\,s^{-2}}$; use the $g = 10$ final answers where the solutions give them.

Questions

Question 1	*10 marks*	2
Question 2	*11 marks*	3
Question 3	*9 marks*	4
Question 4	*11 marks*	5
Question 5	*12 marks*	6
Question 6	*8 marks*	7
Question 7	*7 marks*	8
Question 8	*7 marks*	9
Question 9	*7 marks*	10
Question 10	*9 marks*	11
Question 11	*11 marks*	12
Question 12	*12 marks*	13
Question 13	*10 marks*	14
Question 14	*11 marks*	15
Question 15	*11 marks*	16

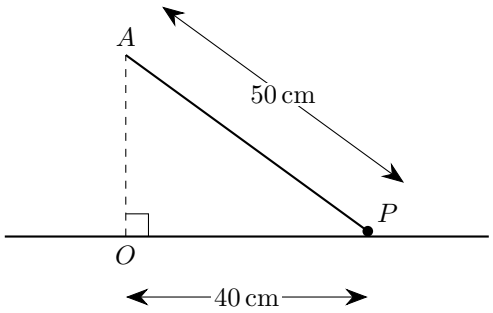
1. A cyclist rides round a circular track which is banked at a constant angle to the horizontal.
The cyclist and bicycle are modelled as a particle moving at constant speed in a horizontal circle, taking 22 s to complete each lap.
At this speed there is no sideways frictional force on the bicycle, and the magnitude of the normal reaction of the track on the particle is $\frac{25}{24}$ times the weight of the particle.

(a) Find the radius of the circle. **[7]**

(b) Identify one limitation of this model. **[1]**

The cyclist now slows down, so that each lap of the same horizontal circle takes 30 s, again at constant speed. The cyclist and bicycle are still modelled as a particle.

(c) Describe the extra force that will now be acting on the particle, stating its direction and giving a reason for this direction. **[2]**



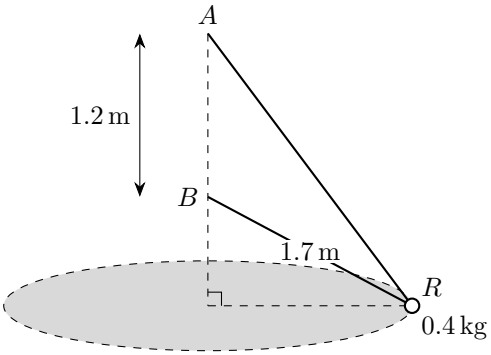
2. A particle P of mass 0.8 kg is attached to one end of a light inextensible string of length 50 cm. The other end of the string is attached to a fixed point A that is vertically above the point O on a smooth horizontal table.
The particle remains in contact with the table, with the string taut, and moves with constant speed in a horizontal circle with centre O and radius 40 cm, as shown in the diagram above.
The tension in the string and the normal reaction between P and the table are equal in magnitude.

(a) (i) Find the tension in the string. **[3]**

(ii) Find the speed of P. **[3]**

The string will break if the tension in it exceeds 15 N. The speed of P is now gradually increased.

(b) Show that P loses contact with the table before the string breaks, and find the speed of P at the instant it loses contact. **[5]**



3. The diagram above shows a light inextensible string of length 4.2 m passing through a small smooth ring R of mass 0.4 kg. The ends of the string are attached to fixed points A and B, where A is 1.2 m vertically above B.
The ring moves with constant speed in a horizontal circle whose centre lies on the line through A and B, vertically below B. The distance BR is 1.7 m and both sections of the string are taut.

(a) Show that the radius of the circle is 1.5 m. **[2]**

(b) Find the tension in the string. **[3]**

(c) Find the time taken for R to complete one revolution. **[4]**

4. A particle moves with constant speed on a circular path of radius $2\sqrt{2}\,\text{m}$ in a horizontal plane.
The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular and lie in this plane.
The centre of the circle has position vector $(3\mathbf{i} + 3\mathbf{j})\,\text{m}$ relative to the origin.
At time $t = 0$, the particle is at the point with position vector $(5\mathbf{i} + \mathbf{j})\,\text{m}$ and its velocity is $-\frac{\pi}{2}(\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$.

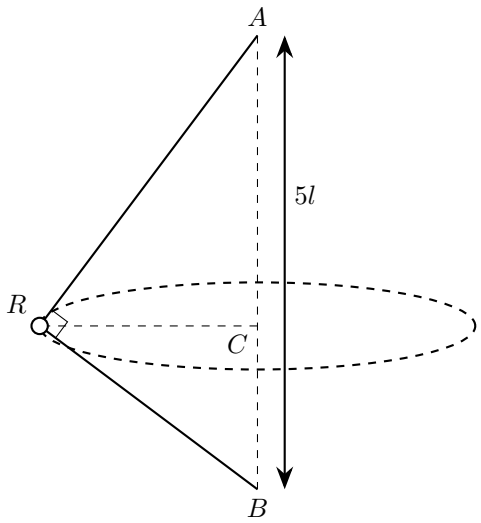
(a) Calculate the angular speed of the particle. **[2]**

(b) Find an expression for the position vector of the particle at time t seconds. **[3]**

(c) Find an expression for the acceleration of the particle at time t seconds. **[3]**

(d) State the magnitude of the acceleration of the particle. **[1]**

(e) Find the time when the acceleration of the particle is first directed towards the origin. **[2]**

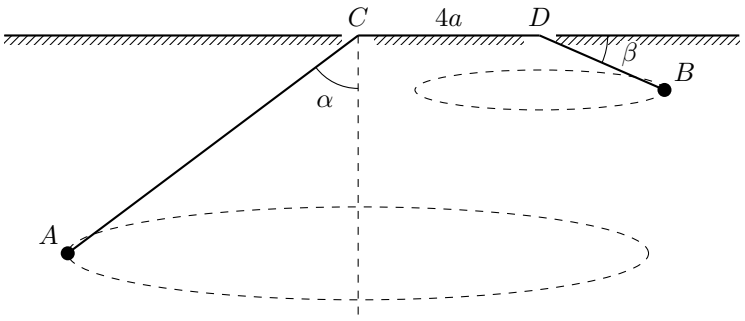


5. One end of a light inextensible string of length $7l$ is attached to a fixed point A. A small smooth ring R of mass m is threaded on the string, and the other end of the string is attached to a fixed point B, vertically below A, with $AB = 5l$.
The ring moves with constant speed in a horizontal circle whose centre C lies on AB. Both sections of the string are taut and angle $ARB = 90°$, as shown in the diagram above.

(a) Show that AR is either $3l$ or $4l$. **[2]**

(b) By resolving vertically for the ring, show that $AR = 4l$ and find, in terms of m and g, the tension in the string. **[5]**

(c) Find, in terms of g and l, the time taken for the ring to complete one revolution. **[5]**

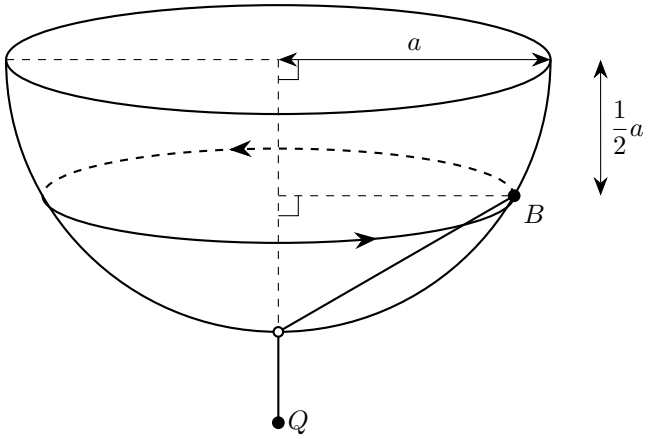


6. A light inextensible string AB of length $15a$ passes through two small holes C and D in a smooth horizontal table, where $CD = 4a$.
A particle of mass $3m$ is attached at the end A and moves in a horizontal circle, centred vertically below C, with constant angular speed ω.
A particle of mass $2m$ is attached at the end B and moves in a horizontal circle, centred vertically below D, with constant angular speed 2ω.
AC makes an angle α with the downward vertical, where $\cos\alpha = \frac{3}{5}$, and DB makes an angle β with the horizontal, as shown in the diagram above.

(a) Find the lengths AC and DB in terms of a. **[4]**

(b) Find ω in terms of a and g, and find the vertical distance of B below the table in terms of a. **[4]**

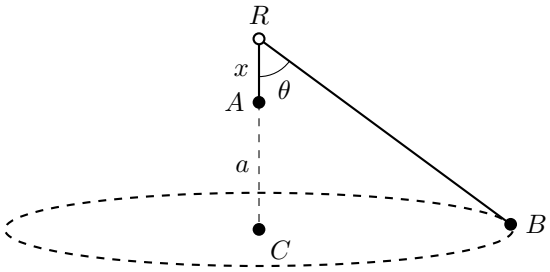
7. A hollow hemispherical bowl of radius a has a rough inner surface and is fixed with its axis vertical.
A particle P of mass m moves with constant speed $\sqrt{\frac{6}{5}ga}$ in a horizontal circle on the inner surface of the bowl, at a height $\frac{1}{5}a$ above the lowest point of the bowl.
The coefficient of friction between P and the bowl is μ, and P is on the point of slipping up the surface of the bowl.
Find the value of μ. **[7]**



8. A hemispherical shell of radius a is fixed with its rim uppermost and horizontal. There is a small smooth hole at the lowest point of the shell.
A small bead, B, of mass m, is moving with constant angular speed, ω, in a horizontal circle on the smooth inner surface of the shell. The centre of the path of B is at a distance $\frac{1}{2}a$ vertically below the level of the rim of the hemisphere, as shown in the diagram above.
A light inextensible string is attached to B and passes through the hole. A particle, Q, of mass $2m$ is attached to the other end of the string and hangs at rest vertically below the hole.

(a) Find, in terms of m and g, the magnitude of the normal reaction of the shell on B. **[4]**

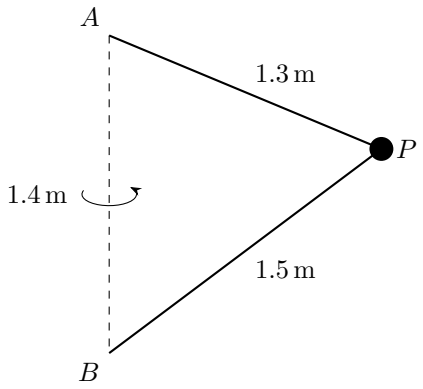
(b) Find the magnitude of ω, giving your answer in terms of a and g. **[3]**



9. A light inextensible string of length $3a$ is threaded through a fixed smooth ring R.
One end of the string is attached to a particle A of mass $5m$, and the other end is attached to a particle B of mass $3m$.
The particle A hangs in equilibrium at a distance x vertically below the ring, and the angle between AR and BR is θ, as shown in the diagram above.
The particle B moves with constant speed in a horizontal circle whose centre C is a distance a vertically below A.

(a) Show that $\cos\theta = \frac{3}{5}$ and find x in terms of a. **[4]**

(b) Find the speed of B in terms of a and g. **[3]**



10. A particle P of mass 0.7 kg is attached to one end of a light inextensible string of length 1.3 m. The other end of this string is attached to the fixed point A.
A second light inextensible string, of length 1.5 m, has one end attached to P and the other end attached to the fixed point B. The point B is vertically below A with $AB = 1.4$ m, as shown in the diagram above. The particle moves in a horizontal circle with constant angular speed ω rad s^{-1}, with both strings taut.

(a) Show that the tension in the string BP is $\frac{3}{8}\left(\omega^2 - 2g\right)$ N, where g m s^{-2} is the acceleration due to gravity. **[7]**

(b) The string BP will break if the tension in it exceeds 15 N.
Hence find the range of values of ω for which the tension in BP is positive and BP does not break. **[2]**

11. A light inextensible string has length $35a$. One end of the string is attached to a fixed point A and the other end is attached to a fixed point B, where A is vertically above B and $AB = 7a$.
A small ball of mass m is attached to a point P on the string, where $AP = 20a$.
The ball moves in a horizontal circle with constant speed v, with both AP and BP taut.
The string will break if the tension in either part of it exceeds $\frac{10mg}{7}$.
Model the ball as a particle and assume that the string does not break.

(a) Show that

$$9ag < v^2 \leq 15ag$$

[8]

(b) Find, in terms of a and g, the time taken for the ball to make one complete revolution when the tensions in AP and BP are equal. **[3]**

12. In this question use $g = 9.81\,\mathrm{m\,s^{-2}}$.
A conical pendulum is made from an elastic string and a sphere of mass $0.5\,\mathrm{kg}$.
The string has natural length $2\,\mathrm{m}$ and modulus of elasticity $100\,\mathrm{N}$.
The sphere describes a horizontal circle at a constant speed of $5\,\mathrm{m\,s^{-1}}$.
The angle between the elastic string and the vertical is α.

(a) Show that

$$25\cos^2\alpha = 9.81\sin^2\alpha\,(2\cos\alpha + 0.0981)$$ **[7]**

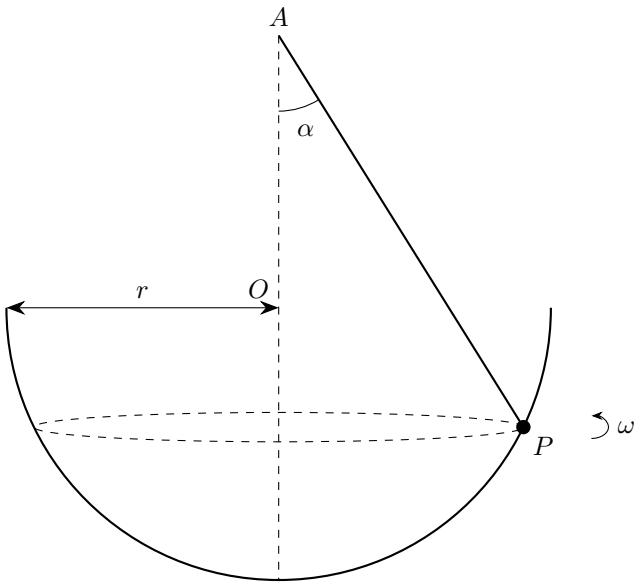
(b) Use your calculator to find α, giving your answer in degrees. **[1]**

(c) Find the elastic potential energy stored in the string. **[4]**

13. A hollow right circular cone C of height 3 m and base radius 4 m is fixed with its axis vertical and its vertex downwards.
One end of a light elastic string of natural length 3 m and modulus of elasticity 16 N is fixed to the vertex of C. The other end of the string is attached to a particle P of mass 5 kg.
P moves in a horizontal circle with constant speed and in contact with the smooth inner surface of C.

(a) Find the greatest speed of P for which the tension in the string is zero. **[3]**

(b) The speed of P is now $6\,\mathrm{m\,s^{-1}}$.
Find the extension of the string and the magnitude of the normal reaction between P and C. **[7]**



14. The diagram above shows a hemispherical bowl. The rim of this bowl is a circle with centre O and radius r. The bowl is fixed with its rim horizontal and uppermost.
The point A is fixed vertically above O, with $OA = r$. A particle P, of mass m, is connected to A by a light inextensible string and describes a horizontal circle with constant angular speed ω on the smooth inner surface of the bowl.
The string is taut, and AP makes an angle α with the vertical, where $0 < \alpha < \frac{\pi}{4}$.

(a) Show that the tension in the string is $2m\cos\alpha\left(g - r\omega^2\cos 2\alpha\right)$. **[7]**

(b) Find, in terms of g, r and α, the range of values of ω^2 for which this motion is possible, with the string taut and P in contact with the bowl. **[4]**

15. A horizontal turntable rotates with constant angular speed $\omega\,\text{rad}\,\text{s}^{-1}$ about a fixed vertical axis through its centre O.
One end of a light elastic string of natural length $l\,\text{m}$ and modulus of elasticity $\lambda\,\text{N}$ is attached to O. The other end of the string is attached to a particle A of mass $m\,\text{kg}$, which rests on the turntable.
The coefficient of friction between A and the turntable is μ.
The particle A moves with the turntable, with the string horizontal and taut, and A is on the point of slipping away from O. The extension of the string is denoted by $x\,\text{m}$.

(a) Show that

$$x = \frac{ml\left(l\omega^2 - \mu g\right)}{\lambda - ml\omega^2}$$ **[3]**

(b) Using a binomial series, show that if λ is very large then

$$\lambda x \approx ml\left(l\omega^2 - \mu g\right)\left(1 + \frac{ml\omega^2}{\lambda}\right) \approx ml\left(l\omega^2 - \mu g\right)$$ **[3]**

(c) By finding an approximation for the tension in the string, explain how the result obtained when λ is very large relates to the situation when the string is inextensible and of length $l\,\text{m}$. **[1]**

The nature of the surface of the turntable is such that the modelling assumption that μ is constant is justifiable provided that the angular speed does not exceed $12\,\text{rad}\,\text{s}^{-1}$.
In the case where $m = 0.25$, $l = 0.5$, $\lambda = 400$ and $\mu = 0.4$, the extension of the string is measured as being $2.0\,\text{cm}$.

(d) Use the approximation $\lambda x \approx ml\left(l\omega^2 - \mu g\right)$ to estimate the value of ω. **[2]**

(e) By comparing the radius used in part **(d)** with the true radius of the circle, explain whether your estimate is greater or less than the true value of ω, and hence whether the modelling assumption is necessarily justifiable in this situation. **[2]**