FURTHER MECHANICS / CIRCULAR MOTION & OSCILLATIONS

Simple Harmonic Motion

Mark scheme 10 questions · 139 marks

EXAM BOARDS Edexcel FM2 9FM0/4C · OCR MEI Mech Major Y421

Questions

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	Total	**139**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$\dot{x} = -\frac{3}{4}\pi \sin\left(\frac{1}{4}\pi t\right) + \frac{3\sqrt{3}}{4}\pi \cos\left(\frac{1}{4}\pi t\right)$ $\Rightarrow \ddot{x} = \ldots$	**M1**	Differentiates their expression for x twice, each term changing between sine and cosine. Condone sign and coefficient slips.
	$\ddot{x} = -\frac{3}{16}\pi^2 \cos\left(\frac{1}{4}\pi t\right) - \frac{3\sqrt{3}}{16}\pi^2 \sin\left(\frac{1}{4}\pi t\right)$	**A1**	
	$\ddot{x} = -\frac{\pi^2}{16}\left(3\cos\left(\frac{1}{4}\pi t\right) + 3\sqrt{3}\sin\left(\frac{1}{4}\pi t\right)\right)$ $= -\frac{\pi^2}{16}x$, so simple harmonic motion	**A1 AG**	Correct $\ddot{x}$ shown equal to $-\frac{\pi^2}{16}x$ without error, with a conclusion of simple harmonic motion. A0 if the conclusion is missing.
		[3]	
1(b)	$T = \frac{2\pi}{\text{“}\frac{\pi}{4}\text{”}} = \ldots$	**M1**	Uses $T = \frac{2\pi}{\omega}$ with their ω, from $\ddot{x} = -\omega^2 x$ or the argument $\frac{1}{4}\pi t$.
	$T = 8\,\text{s}$	**A1**	
		[2]	
1(c)	$a = \sqrt{3^2 + \left(3\sqrt{3}\right)^2} = \ldots$	**M1**	Uses $\sqrt{p^2 + q^2}$ with the coefficients 3 and $3\sqrt{3}$, or finds the greatest value of x by another valid method. Condone a squaring slip.
	$a = 6\,\text{m}$	**A1**	
		[2]	
1(d)	$v_{\max} = \text{“6”} \times \text{“}\frac{\pi}{4}\text{”} = \ldots$	**M1**	Uses $a\omega$ with their amplitude and ω, or maximises their $\dot{x}$ by another valid method.
	$v_{\max} = \frac{3\pi}{2}\,\text{m}\,\text{s}^{-1}$	**A1**	oe in terms of π.
		[2]	
1(e)	$\left(\frac{\pi}{4}\right)^2 \left(\text{“6”}^2 - x^2\right) > \left(\frac{3\sqrt{3}}{4}\pi\right)^2$ $\Rightarrow x^2 < \ldots$	**M1**	Uses $v^2 = \omega^2\left(a^2 - x^2\right)$ with their a and ω and the given speed, as an equation or inequality, to find x, or sets their $\dot{x}$, or $a\omega\cos\omega t$, equal to the given speed. Condone slips.
	$-3 < x < 3$	**A1**	Boundary $x = \pm 3$, or $\lvert x\rvert < 3$, soi; or, from the speed, a correct cosine or sine value, such as $\cos\frac{1}{4}\pi t = \frac{\sqrt{3}}{2}$, or a correct boundary time.

Question 1 continued

Question	Scheme	Marks	Notes
	$6\sin\left(\frac{1}{4}\pi t\right) = \text{“3”} \Rightarrow t = \ldots$	**M1**	Sets a sine or cosine expression for x, the original included, equal to their boundary displacement, or one for the speed equal to the given speed, and solves for a time.
	$t = \frac{2}{3}$	**A1**	Time $\frac{2}{3}$ from O to $x = 3$, or a correct equivalent time from another form of x or from the speed.
	$4 \times \text{“}\frac{2}{3}\text{”} = \ldots$	**M1**	Uses symmetry to combine their times into the total per oscillation, as four times the time from O to $x = 3$ or an equivalent sum or difference.
	$\frac{8}{3}$ s	**A1**	$\frac{8}{3}$ or awrt 2.67.
		[6]	
Total for Question 1		**15**	

Question	Scheme	Marks	Notes
2(a)	$\frac{2\pi}{\omega} = \frac{\pi}{2} \Rightarrow \omega = 4$	**M1 A1**	**M1** Uses $T = \frac{2\pi}{\omega}$ with $T = \frac{\pi}{2}$ to find ω.
	$1.6^2 = \text{“4”}^2\left(a^2 - 0.3^2\right)$	**M1**	Uses $v^2 = \omega^2\left(a^2 - x^2\right)$ with $x = 0.3$, $v = 1.6$ and their ω. Condone slips in substitution.
	$2.56 = 16\left(a^2 - 0.09\right)$ $a^2 = 0.25 \Rightarrow a = 0.5$	**A1 AG**	Correct equation with $\omega = 4$ solved without error to $a = 0.5$; combined steps accepted. Verifying with $a = 0.5$ scores A0.
		[4]	
2(b)	$1^2 = \text{“4”}^2\left(0.5^2 - x^2\right) \Rightarrow x = \ldots$	**M1**	Uses $v^2 = \omega^2\left(a^2 - x^2\right)$ with $v = 1$, $a = 0.5$ and their ω to find x, as an equation or inequality; or sets $a\omega\cos\omega t$ or $a\omega\sin\omega t$ equal to 1.
	$x = \frac{\sqrt{3}}{4}$	**A1**	oe, or awrt 0.433. Sign not needed. Or, from the speed, $\cos 4t = \frac{1}{2}$, $\sin 4t = \frac{1}{2}$ or a correct boundary time.
	$0.5\sin 4t = \text{“}\frac{\sqrt{3}}{4}\text{”} \Rightarrow t = \ldots$	**M1**	Uses $x = a\sin\omega t$ or $x = a\cos\omega t$, with their ω, equated to their boundary displacement, or its speed equated to 1, and solves for t.
	$t = \frac{\pi}{12}$	**A1**	oe, including $t = \frac{\pi}{24}$ from an end point with $x = a\cos\omega t$.
	$4 \times \text{“}\frac{\pi}{12}\text{”} = \frac{\pi}{3}\,\text{s}$	**M1 A1**	**M1** Multiplies their time from O by 4, or uses an equivalent symmetry argument, such as $4\left(\frac{\pi}{8} - \text{“}\frac{\pi}{24}\text{”}\right)$ from an end point.
		[6]	
2(c)	$\frac{1}{2} \times 0.5 \times v^2 = 0.36 \Rightarrow v^2 = 1.44$ $1.44 = \text{“4”}^2\left(0.25 - x^2\right) \Rightarrow x = \ldots$	**M1**	Uses $\frac{1}{2}mv^2 = 0.36$ for v^2, then $v^2 = \omega^2\left(a^2 - x^2\right)$ with $a = 0.5$ and their ω to find x. Condone slips.
	$x = 0.4$	**A1**	Accept ± 0.4.
	$F = 0.5 \times \text{“4”}^2 \times \text{“0.4”}$	**M1**	Uses Newton's second law with acceleration of magnitude $\omega^2 x$, with their ω and displacement. Not for the acceleration alone.

Question 2 continued

Question	Scheme	Marks	Notes
	$F = 3.2\,\text{N}$	**A1**	
		[4]	
	Total for Question 2	**14**	

Question	Scheme	Marks	Notes
3(a)	$3.6^2 = \omega^2\left(a^2 - 0.5^2\right)$ $1.5^2 = \omega^2\left(a^2 - 1.2^2\right)$	**M1 A1**	**M1** Uses $v^2 = \omega^2\left(a^2 - x^2\right)$ at both given positions. Condone a slip in one equation. **A1** Both equations correct.
	$10.71 = 1.19\,\omega^2 \Rightarrow \omega^2 = 9$	**dM1 A1**	**dM1** Solves their two equations together, by subtracting or dividing, eliminating a or ω. **A1** $\omega^2 = 9$ or $\omega = 3$; or $a^2 = 1.69$ where ω is eliminated.
	$12.96 = 9\left(a^2 - 0.25\right) \Rightarrow a^2 = 1.69$ $a = 1.3$	**A1 AG**	Correct equations leading without error to $a = 1.3$; combined steps accepted. Not for substituting $a = 1.3$ to verify.
		[5]	
3(b)	Period $= \frac{2\pi}{3}$ s	**B1**	
	$\omega^2 a = 9 \times 1.3 = 11.7\,\mathrm{m\,s^{-2}}$	**M1 A1**	**M1** Uses $\omega^2 a$ with their ω and $a = 1.3$. Not for ωa. **A1** 11.7 cao.
		[3]	
3(c)	$9\lvert x\rvert \geq 6.3 \Rightarrow \lvert x\rvert \geq 0.7$	**M1 A1**	**M1** Sets $\omega^2 x$, with their ω, equal to 6.3, as an equation or inequality. **A1** 0.7, or $\cos 3t$ or $\sin 3t = \frac{7}{13}$, seen; the inequality is not needed.
	$1.3\cos 3t = 0.7$ $t = \frac{1}{3}\arccos\frac{7}{13} \approx 0.334$	**M1 A1**	**M1** Equates $1.3\cos\omega t$ or $1.3\sin\omega t$, with their ω, to their 0.7, or uses their value of $\cos 3t$ or $\sin 3t$, and solves for t. **A1** $\frac{1}{3}\arccos\frac{7}{13}$ or awrt 0.334; or the time from O, $\frac{1}{3}\arcsin\frac{7}{13}$ or awrt 0.190.
	$T = 4 \times$ "$\frac{1}{3}\arccos\frac{7}{13}$" $= \frac{4}{3}\arccos\frac{7}{13} \approx 1.34$	**M1 A1**	**M1** Multiplies their time from an extreme position by 4, or subtracts 4 times their time from O from their period. **A1** Exact form oe, or awrt 1.34.
		[6]	
Total for Question 3		**14**	

Question	Scheme	Marks	Notes
4(a)	(centre =) 7.4, $a = 3.8$	**B1**	Both values.
	$\frac{1}{2}T = 6.25 \Rightarrow \omega = \frac{2\pi}{12.5} = \ldots$	**M1**	Uses 6 hours 15 minutes as half a period in $T = \frac{2\pi}{\omega}$. Condone working in minutes.
	$\omega = \frac{4\pi}{25}$	**A1**	oe, or awrt 0.503 rad per hour, or the equivalent in minutes.
	$1.5^2 = \left(\text{“}\frac{4\pi}{25}\text{”}\right)^2 \left(\text{“}3.8\text{”}^2 - x^2\right)$ $\Rightarrow x = \ldots$	**M1**	Substitutes $v = 1.5$, their a and their ω into $v^2 = \omega^2(a^2 - x^2)$, or uses velocity from $x = a\cos\omega t$, and solves for x. Units consistent.
	$x = \pm 2.35$	**A1**	awrt 2.35, or x^2 awrt 5.53; sign not needed.
	$7.4 \pm 2.35 \Rightarrow 9.75\,\text{m}$ and $5.05\,\text{m}$	**A1**	Both, awrt 9.75 and 5.05.
		[6]	
4(b)	$3.8\cos\left(\text{“}\frac{4\pi}{25}\text{”}\,t\right) = 5.5 - 7.4$	**M1**	Sets a cosine model from high tide, or an equivalent model, equal to depth 5.5, using their centre, amplitude and ω. Condone a sign slip.
	$\cos\left(\frac{4\pi t}{25}\right) = -\frac{1}{2}$	**A1**	Or one correct boundary time, $t = \frac{25}{6}$ or $\frac{25}{3}$ hours after 05:40, oe.
	$\frac{4\pi t}{25} = \frac{2\pi}{3}, \frac{4\pi}{3} \Rightarrow t = \frac{25}{6}, \frac{25}{3}$ $\frac{25}{3} - \frac{25}{6} = \ldots$	**dM1**	Finds two boundary times within one period from their equation and subtracts, or doubles the time from one boundary to low tide.
	$\frac{25}{6}$ hours (4 hours 10 minutes)	**A1**	Or awrt 4.17 hours, or 250 minutes.
		[4]	
	Total for Question 4	**10**	

Question	Scheme	Marks	Notes
5(a)	$v\frac{\mathrm{d}v}{\mathrm{d}x} = 9(2-x)$	**M1**	Uses $v\frac{\mathrm{d}v}{\mathrm{d}x}$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right)$ for the acceleration in the given equation; or identifies the centre $x=2$ and $\omega=3$ from it.
	$\int v\,\mathrm{d}v = \int(18-9x)\,\mathrm{d}x$ $\frac{1}{2}v^2 = 18x - \frac{9}{2}x^2 + c$	**M1 A1**	**M1** Separates the variables and integrates both sides, the power of at least one term increased by 1; or uses $v^2=\omega^2\left(a^2-(x-2)^2\right)$ with their ω. Condone sign slips. **A1** Correct, oe; the constant, or the amplitude, need not be seen yet.
	$0 = 90 - \frac{225}{2} + c \Rightarrow c = \ldots$	**dM1**	Substitutes $x=5$, $v=0$ into their integrated equation to find the constant, or uses them as limits; or takes the amplitude as the distance from their centre to $x=5$.
	$c = \frac{45}{2}$ $v^2 = 36x - 9x^2 + 45$ $\quad = 9(5-x)(x+1)$	**A1 AG**	Correct constant, or amplitude 3, and equation for v^2, leading without error to the printed result; combined steps accepted.
		[5]	
5(b)	$v^2 = 9\left(9-(x-2)^2\right)$ $\frac{\mathrm{d}x}{\mathrm{d}t} = 3\sqrt{9-(x-2)^2}$ $\int \frac{1}{\sqrt{9-(x-2)^2}}\,\mathrm{d}x = \int 3\,\mathrm{d}t$	**M1**	Completes the square as $p-(x-q)^2$, takes the positive root and separates the variables; or finds the centre and amplitude from part (a). Condone slips in p, q.
	$\arcsin\left(\frac{x-2}{3}\right) = 3t + K$	**M1 A1**	**M1** Integrates to the form $\arcsin\left(\frac{x-q}{r}\right) = kt + K$, or writes $x = q + r\sin(kt+K)$ with their centre and amplitude. **A1** Correct, or $x = 2 + 3\sin(3t+K)$, oe.
	$K = \arcsin\left(\frac{\frac{1}{2}-2}{3}\right) = \ldots$	**dM1**	Substitutes $t=0$, $x=\frac{1}{2}$ into their equation to find the constant.
	$K = -\frac{\pi}{6}$	**A1**	On the $x = 2+3\sin(3t+K)$ route, needs $\dot{x}>0$ at $t=0$ used to reject $-\frac{5\pi}{6}$.

Question 5 continued

Question	Scheme	Marks	Notes
	$\arcsin\left(\frac{x-2}{3}\right) = 3t - \frac{\pi}{6}$ $x = 2 + 3\sin\left(3t - \frac{\pi}{6}\right)$	**A1 AG**	Correct equation with $K = -\frac{\pi}{6}$, rearranged without error to the printed result.
		[6]	
	Total for Question 5	**11**	

Question	Scheme	Marks	Notes
6(a)	$R - mg = -m\omega^2 x$	**M1**	Forms an equation of motion for the box with reaction, weight and acceleration of magnitude $\omega^2 x$, or $\omega^2 a$ at an extreme point. Condone sign slips.
	Greatest $R = m\left(g + a\omega^2\right)$ Least $R = m\left(g - a\omega^2\right)$	**A1**	Both correct.
	$3m\left(g + a\omega^2\right) = 7m\left(g - a\omega^2\right)$ $\Rightarrow \omega^2 = \frac{2g}{5a}$	**A1 AG**	Correct equation from the $\frac{7}{3}$ ratio, leading without error to the printed result.
		[3]	
6(b)	$R = m\left(g - \frac{2gx}{5a}\right) = 0$ $\Rightarrow x = \ldots$	**M1**	Sets their reaction from (a) to zero, or the downward acceleration $\omega^2 x$ equal to g, with $\omega^2 = \frac{2g}{5a}$, and solves for x.
	$x = \frac{5a}{2}$	**A1**	
	$\frac{5a}{2} < 3a$, so the box leaves the platform	**E1**	Compares $\frac{5a}{2}$ with the amplitude $3a$, or $3a\omega^2$ with g, and concludes the box leaves the platform.
		[3]	
6(c)	$v^2 = \frac{2g}{5a}\left(9a^2 - \frac{25a^2}{4}\right)$	**M1 A1**	**M1** Uses $v^2 = \omega^2\left(A^2 - x^2\right)$ with $A = 3a$ and their height from (b). Not for amplitude a. **A1** Correct substituted expression.
	$v = \sqrt{\frac{11ga}{10}}$	**A1**	oe, such as $\sqrt{1.1ga}$.
		[3]	
	Total for Question 6	**9**	

Question	Scheme	Marks	Notes
7(a)	$\frac{36e_0}{0.9} = 0.4 \times 10 \Rightarrow e_0 = 0.1$	**B1**	
	$0.4\ddot{x} = 0.4 \times 10 - \frac{36(0.1+x)}{0.9}$	**M1 A1**	**M1** Forms Newton's second law with weight and Hooke's-law tension, the extension in terms of a displacement variable. Condone sign slips. **A1** Correct equation, with displacement measured from any point.
	$\ddot{x} = -100x$, so simple harmonic motion	**A1**	Needs x measured from equilibrium, or a substitution reaching this form, and a conclusion of simple harmonic motion.
	period $= \frac{2\pi}{\omega} = \frac{2\pi}{10} = \frac{\pi}{5}$	**M1** **A1 AG**	**M1** Uses $\frac{2\pi}{\omega}$ with their ω from an equation of the form $\ddot{x} = -\omega^2 x$. **A1 AG** $\omega = 10$ from a correct equation of motion, reaching $\frac{\pi}{5}$ without error.
		[6]	
7(b)	$3 = 10^2\left(a^2 - 0.1^2\right) \Rightarrow a = \ldots$	**M1**	Uses $v^2 = \omega^2\left(a^2 - x^2\right)$ with $v = \sqrt{3}$, their ω and their equilibrium extension as x, or an energy equation from B to the slack point with EPE, GPE and KE. Condone the sign of x.
	$a = 0.2$	**A1**	Or the extension at B, 0.3.
	$AB = 0.9 + 0.1 + 0.2 = 1.2\,\text{m}$	**A1**	
		[3]	
7(c)	$0 = 3 - 2 \times 10s \Rightarrow s = \ldots\ [= 0.15]$	**M1**	Uses $v^2 = u^2 + 2\alpha s$, or energy, for the free motion from the slack point, with $v = 0$ and acceleration ± 10. Condone sign slips.
	$BC = 0.2 + 0.1 + 0.15 = 0.45\,\text{m}$	**A1**	
	$0.2\cos 10t = -0.1 \Rightarrow t = \ldots$	**M1**	Sets their $a\cos 10t$, or an equivalent sine form with any phase or time added, equal to the slack displacement and solves for t. Condone the sign of x.
	$10t = \frac{2\pi}{3} \Rightarrow t = \frac{\pi}{15}$	**A1**	

Question 7 continued

Question	Scheme	Marks	Notes
	$0 = \sqrt{3} - 10t_2 \Rightarrow t_2 = \ldots$ time = "$\frac{\pi}{15}$" + $\frac{\sqrt{3}}{10}$	**M1**	Uses $v = u + \alpha t$, or equivalent, with $u = \sqrt{3}$, $v = 0$ and acceleration ± 10, and adds their time to the slack point.
	$= \frac{2\pi+3\sqrt{3}}{30}$ s	**A1**	Any exact equivalent, such as $\frac{\pi}{15} + \frac{\sqrt{3}}{10}$.
		[6]	
	Total for Question 7	**15**	

Question	Scheme	Marks	Notes
8(a)	$\frac{3mge}{5a} = mg\sin\alpha \Rightarrow e = \ldots$	**M1**	Resolves parallel to the plane at A with a Hooke's law tension and a weight component. Condone sin/cos confusion.
	$e = a$	**A1**	
	$T = \frac{3mg(a-x)}{5a}$	**B1**	oe, such as $\frac{3mg}{5} - \frac{3mgx}{5a}$.
	$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{3mg(a-x)}{5a} - mg\sin\alpha$	**M1 A1**	**M1** Forms an equation of motion along the plane with a tension in terms of x and the weight component. Condone sign slips. **A1** Correct equation, need not be simplified.
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{3gx}{5a} = 0$	**A1 AG**	Correct equation of motion leading without error to the printed equation; combined simplification accepted.
		[6]	
8(b)	$R\sqrt{\frac{3g}{5a}} = 2\sqrt{\frac{3ag}{5}} \Rightarrow R = \ldots$	**M1**	Uses maximum speed $R\omega$, with $\omega^2 = \frac{3g}{5a}$, equal to the projection speed, or $v^2 = \omega^2(R^2 - x^2)$ at $x = 0$.
	$R = 2a$	**A1**	
	"$2a$" $\sin\omega t =$ "a" $\Rightarrow \omega t = \ldots$	**M1**	Sets a displacement of the form $R\sin\omega t$, with their R, equal to their extension at A, or uses an equivalent cosine form with its phase.
	$\omega t = \frac{\pi}{6}$	**A1**	
	$t = \frac{\pi}{6}\sqrt{\frac{5a}{3g}}$	**A1**	oe, such as $\frac{\pi}{18}\sqrt{\frac{15a}{g}}$.
		[5]	
8(c)	$v^2 = \frac{3g}{5a}\left("2a"^2 - "a"^2\right)$ $= \ldots \ [= \frac{9ag}{5}]$	**M1**	Uses $v^2 = \omega^2(R^2 - x^2)$ with their R and $x =$ their extension at A, or an energy equation from A including the EPE at A; one such equation to the top earns both M marks.
	$\frac{1}{2}m\left("\frac{9ag}{5}"\right) = mgd\sin\alpha$ $\Rightarrow d = \ldots \ [= \frac{3a}{2}]$	**M1**	Forms an energy equation with KE and GPE, or uses constant acceleration, once the string is slack. Condone sin/cos confusion.

Question 8 continued

Question	Scheme	Marks	Notes
	$a + \frac{3a}{2} = \frac{5a}{2}$	**A1**	oe. Needs the distance from A, not from the slack point.
		[3]	
	Total for Question 8	**14**	

Question	Scheme	Marks	Notes
9(a)	$\frac{3mg(AE-2L)}{2L} = \frac{2mg(6L-AE)}{4L}$	**M1**	Equates two tensions of the form $\frac{\lambda x}{l}$, with both extensions in terms of one unknown length. Condone slips in the extensions.
	$3(AE-2L) = 6L - AE \Rightarrow AE = 3L$	**A1 AG**	Correct equation in one unknown leading without error to $AE = 3L$.
		[2]	
9(b)	$T_{AP} = \frac{3mg(L+y)}{2L}$ $T_{BP} = \frac{2mg(3L-y)}{4L}$	**B1**	Both tensions correct in terms of a displacement from E, measured either way, in any letter.
	$m\ddot{y} = \frac{mg(3L-y)}{2L} - \frac{3mg(L+y)}{2L}$	**M1 A1**	**M1** Forms Newton's second law with mass times acceleration and the difference of their two tensions. Condone sign slips. **A1** Correct equation, signs consistent with their direction of displacement.
	$\ddot{y} = -\frac{2g}{L}y \Rightarrow$ simple harmonic motion, centre E	**A1**	Correct equation with a conclusion of simple harmonic motion about E, or where $y = 0$. Strings staying taut need not be shown.
	Period $= \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\text{“}\frac{2g}{L}\text{”}}}$	**M1**	Uses $\frac{2\pi}{\omega}$ with their ω from an equation of the form $\ddot{y} = -\omega^2 y$.
	$= 2\pi\sqrt{\frac{L}{2g}} = \pi\sqrt{\frac{2L}{g}}$	**A1 AG**	From a correct $\omega^2 = \frac{2g}{L}$, reaching the printed period without error.
		[6]	
9(c)	$y = \frac{5}{2}L - 3L = -\frac{1}{2}L$	**B1**	Either sign.
	$v^2 = \frac{2g}{L}\left(\frac{4L^2}{9} - \frac{L^2}{4}\right)$	**M1**	Uses $v^2 = \omega^2(a^2 - y^2)$ with their ω^2, $a = \frac{2}{3}L$ and their displacement, or an equivalent energy equation. Not for quoting the formula alone.
	$v = \sqrt{\frac{7gL}{18}}$	**A1**	Or $\frac{\sqrt{14gL}}{6}$ oe, or awrt $0.624\sqrt{gL}$.
		[3]	

Question 9 continued

Question	Scheme	Marks	Notes
9(d)	$\frac{3mg(L+y)}{2L} = 2mg \Rightarrow y = \ldots$	**M1**	Sets their tension in AP equal to, or greater than, $2mg$ and solves for the displacement or for AP.
	$y = \frac{1}{3}L$	**A1**	Or $AP = \frac{10}{3}L$.
	$y = -\frac{2}{3}L\cos\omega t$ $-\frac{2}{3}L\cos\omega t = \text{“}\frac{1}{3}L\text{”} \Rightarrow \cos\omega t = \ldots$	**M1**	Uses a cosine or sine model with amplitude $\frac{2}{3}L$ and their boundary displacement to form an equation in ωt. Condone sign slips.
	$\cos\omega t = -\frac{1}{2} \Rightarrow \omega t = \frac{2\pi}{3}$	**A1**	Or a correct boundary value of ωt for their model.
	$\left(\frac{4\pi}{3} - \frac{2\pi}{3}\right)\sqrt{\frac{L}{2g}} = \ldots$	**dM1**	Finds the interval in ωt in one oscillation for which the tension exceeds $2mg$, and divides by their ω.
	$\frac{\pi}{3}\sqrt{\frac{2L}{g}}$	**A1**	Any exact equivalent, such as $\frac{2\pi}{3}\sqrt{\frac{L}{2g}}$.
		[6]	
	Total for Question 9	**17**	

Question	Scheme	Marks	Notes
10(a)	$v^2 = \left(\sqrt{gl}\right)^2 + 2gl \Rightarrow v^2 = 3gl$	**M1 A1**	**M1** Uses $v^2 = u^2 + 2as$ with $u = \sqrt{gl}$, $a = g$ and $s = l$. Condone sign slips.
	$\frac{1}{2}m(3gl) + mgd = \frac{mgd^2}{2l}$	**M1 A1**	**M1** Forms an energy equation to the lowest point with KE, GPE and EPE in the extension squared. Condone sign slips. **A1** Correct equation oe. A correct energy equation from O also earns the two marks for v^2.
	$d^2 - 2ld - 3l^2 = 0 \Rightarrow d = 3l$ Greatest length $l + 3l = 4l$	**A1 AG**	Correct energy equation solved without error to $d = 3l$, and the length $4l$ stated. The negative root need not be shown.
		[5]	
10(b)	$m\ddot{x} = mg - \frac{mgx}{l}$	**M1 A1**	**M1** Applies Newton's second law to P with the weight and a tension of the form $\frac{kx}{l}$. Condone sign slips.
	$\ddot{x} + \frac{g}{l}x = g \Rightarrow \ddot{x} + \omega^2 x = g$	**A1 AG**	Correct equation of motion leading without error to the printed result, with $\omega^2 = \frac{g}{l}$ identified.
		[3]	
10(c)	$x = y + l \Rightarrow \ddot{y} + \omega^2(y + l) = g$	**M1**	Substitutes $x = y + \frac{g}{\omega^2}$, or $y + l$, with $\ddot{x} = \ddot{y}$ into the differential equation.
	$\ddot{y} + \omega^2 y = 0$	**A1**	
	$\lambda^2 + \omega^2 = 0 \Rightarrow y = A\cos\omega t + B\sin\omega t$	**M1**	Obtains y as $A\cos\omega t + B\sin\omega t$, or an equivalent general solution; the auxiliary equation may be implied.
	$x = l + A\cos\omega t + B\sin\omega t$	**A1**	oe, with l or $\frac{g}{\omega^2}$ added.
	$\dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t$ $0 = l + A, \quad B\omega = \text{“}\sqrt{3gl}\text{”}$	**M1**	Uses $x = 0$ and $\dot{x} =$ their speed from (a) at $t = 0$ in their general solution and its derivative.
	$A = -l$	**A1**	
	$B = \sqrt{3}\,l$	**A1**	oe, such as $\sqrt{3gl}\sqrt{\frac{l}{g}}$.

Question 10 continued

Question	Scheme	Marks	Notes
	$\sqrt{3}\,l\sin\omega t - l\cos\omega t = R\sin(\omega t - \alpha)$ $R\cos\alpha = \sqrt{3}\,l,\ R\sin\alpha = l \Rightarrow R = \ldots,\ \alpha = \ldots$	**M1**	Uses their A and B to find R and α for any single-sinusoid form. Condone the ratio inverted.
	$R = 2l, \quad \alpha = \frac{\pi}{6}$ $x = l + 2l\sin\left(\omega t - \frac{\pi}{6}\right)$	**A1**	Or the correct values for their form.
	$l + 2l\sin\left(\omega t - \frac{\pi}{6}\right) = 0$ $\Rightarrow \sin\left(\omega t - \frac{\pi}{6}\right) = -\frac{1}{2} \Rightarrow \omega t = \ldots$	**M1**	Sets their $x = 0$ and solves for ωt, from their single sinusoid or by any valid method.
	$\omega t - \frac{\pi}{6} = \frac{7\pi}{6}$	**A1**	Or $\omega t = \frac{4\pi}{3}$. Needs the first root after $t = 0$, not $-\frac{\pi}{6}$.
	$t = \frac{4\pi}{3}\sqrt{\frac{l}{g}}$	**A1**	Any exact equivalent.
		[12]	
	Total for Question 10	**20**	