FURTHER MECHANICS / CIRCULAR MOTION & OSCILLATIONS

Motion in a Vertical Circle

Mark scheme 15 questions · 206 marks

EXAM BOARDS Edexcel FM2 9FM0/4C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Major Y421 CIE 9231 Paper 3

Questions

Question	Marks by part	Marks	Page
Question 1	(a) 3 (b) 3 (c) 4	10	4
Question 2	(a) 3 (b) 1 (c) 4 (d)(i) 2 (d)(ii) 2	12	5
Question 3	(a) 6 (b) 6	12	6
Question 4	(a) 7 (b) 2	9	7
Question 5	(a) 8 (b) 3	11	8
Question 6	(a) 8 (b) 5	13	9
Question 7	(a) 8 (b) 4 (c) 4	16	10
Question 8	(a) 8 (b) 7	15	12
Question 9	(a) 6 (b) 2 (c) 4 (d) 1	13	14
Question 10	(a) 1 (b)(i) 4 (b)(ii) 3 (c) 6	14	15
Question 11	(a) 6 (b) 8 (c) 7	21	17
Question 12	(a) 6 (b) 5 (c) 6	17	19
Question 13	(a) 6 (b) 2 (c) 2 (d) 1	11	21
Question 14	(a) 6 (b) 6 (c) 1	13	22
Question 15	(a) 6 (b) 6 (c) 7	19	24
	Total	**206**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$\frac{1}{2}m \times 7^2 = \frac{1}{2}mv^2 + 9.8m\left(1.25 + 1.25\cos\frac{1}{3}\pi\right)$	**M1 A1**	**M1** Forms an energy equation from the lowest point with KE at both points and GPE for a height using a trigonometric ratio of $\frac{1}{3}\pi$ or $\frac{1}{6}\pi$. Condone sign slips. **A1** Correct equation oe; the height need not be simplified.
	$v^2 = 12.25 \Rightarrow v = 3.5\,\mathrm{m\,s^{-1}}$	**A1**	
		[3]	
1(b)	$\frac{\text{“}3.5\text{”}^2}{1.25} = \ldots$	**M1**	Uses $\frac{v^2}{r}$ with their speed from (a) and $r = 1.25$.
	$9.8\,\mathrm{m\,s^{-2}}$	**A1**	
	Towards O	**B1**	Towards O, or along PO, in words or on a diagram.
		[3]	
1(c)	$-9.8m\sin\frac{1}{3}\pi = ma \Rightarrow a = \ldots$	**M1**	Uses Newton's second law along the tangent with a component of the weight as the only force. Condone sign slips and sin/cos confusion.
	$a = -4.9\sqrt{3}$ $[\approx -8.49]$	**A1**	$4.9\sqrt{3}$ oe, or awrt 8.49, either sign.
	$\sqrt{\text{“}9.8\text{”}^2 + \left(\text{“}4.9\sqrt{3}\text{”}\right)^2} = \ldots$	**M1**	Uses Pythagoras with their radial acceleration from (b) and their tangential component.
	$4.9\sqrt{7} \approx 13.0\,\mathrm{m\,s^{-2}}$	**A1**	$4.9\sqrt{7}$ oe, or awrt 13.0.
		[4]	
	Total for Question 1	**10**	

Question	Scheme	Marks	Notes
2(a)	Drop $= 2.5 - 2.5 \times \frac{3}{5}$ $[= 1]$ $\frac{1}{2}(0.5)(2^2) + 0.5(10)(1)$ $= \frac{1}{2}(0.5)v^2$	**M1 A1**	**M1** Forms an energy equation with KE at both points and GPE for a drop of the form $2.5 - 2.5\cos\alpha$. Condone sign slips. **A1** Correct equation, need not be simplified.
	$v^2 = 24 \Rightarrow v = 2\sqrt{6}\,\text{m s}^{-1}$	**A1 AG**	Correct substituted equation leading without error to the printed speed; working backwards from $2\sqrt{6}$ scores A0.
		[3]	
2(b)	The sphere is modelled as a particle	**B1**	Accept its size negligible, oe. Not for an assumption about the string, peg or air.
		[1]	
2(c)	$T_1 - 0.5 \times 10 = 0.5 \times \frac{24}{2.5}$	**M1**	Resolves radially before the peg with tension, weight and acceleration $\frac{v^2}{r}$, where $r = 2.5$. Condone sign slips.
	$T_1 = 9.8\,\text{N}$	**A1**	
	$T_2 - 0.5 \times 10 = 0.5 \times \frac{24}{1.25}$	**M1**	Resolves radially after the peg with tension, weight and acceleration $\frac{v^2}{r}$, where $r = 1.25$, at the same speed. Condone sign slips.
	$T_2 = 14.6\,\text{N}$	**A1**	
		[4]	
2(d)(i)	$\frac{1}{2}(0.5)(2^2) + 0.5(10)(1) - W$ $= \frac{1}{2}(0.5)(4.6^2)$	**M1**	Forms an energy equation with KE at both points, GPE and a work-done term. Condone sign slips.
	$W = 0.71\,\text{J}$	**A1**	Positive 0.71 as the work against air resistance.
		[2]	
2(d)(ii)	$T - 0.5 \times 10 = 0.5 \times \frac{4.6^2}{1.25}$	**M1**	Resolves radially with tension, weight and acceleration $\frac{v^2}{r}$, where $v = 4.6$ and $r = 1.25$. Condone sign slips.
	$T = 13.464 \approx 13.5\,\text{N}$	**A1**	awrt 13.5
		[2]	
	Total for Question 2	**12**	

Question	Scheme	Marks	Notes
3(a)	$2.5g\cos\frac{1}{3}\pi = \frac{2.5v_1^2}{1.2} \Rightarrow v_1^2 = \ldots$	**M1**	Resolves towards O with $T = 0$, a weight component and mass times $\frac{v^2}{r}$, where $r = 1.2$. Condone sign slips and sin/cos confusion.
	$v_1^2 = 5.88$	**A1**	Or $0.6g$ oe.
	$\frac{1}{2}(2.5)v^2 = \frac{1}{2}(2.5)\text{“}5.88\text{”} + 2.5g(1.8)$	**M1 A1**	**M1** Forms an energy equation from the lowest point to the slack point with KE at both points and GPE. Condone sign slips. **A1** Correct equation.
	$J = 2.5 \times \text{“}\sqrt{41.16}\text{”} - 0$	**M1**	Uses impulse equals mass times their speed at the lowest point from an energy equation. Not for v^2 in place of v.
	$J = \frac{7\sqrt{21}}{2} \approx 16.0$	**A1**	Exact form or awrt 16.0.
		[6]	
3(b)	$\frac{1}{2}(2.5)\text{“}41.16\text{”} = \frac{1}{2}(2.5)v_2^2 + 2.5g(0.6)$	**M1 A1**	**M1** Forms an energy equation from the lowest point or the slack point to $\theta = \frac{1}{3}\pi$, with KE at both points and GPE. Condone sign slips. **A1** Correct equation, from either starting point.
	$v_2^2 = 29.4$	**A1**	Or $3g$ oe.
	$T - 2.5g\cos\frac{1}{3}\pi = \frac{2.5\times\text{“}29.4\text{”}}{1.2}$	**M1 A1**	**M1** Resolves towards O with T, a weight component and mass times $\frac{v^2}{r}$, where $r = 1.2$. Condone sign slips and sin/cos confusion. **A1** Correct equation.
	$T = 73.5\,\text{N}$	**A1**	
		[6]	
	Total for Question 3	**12**	

Question	Scheme	Marks	Notes
4(a)	$v_A = 2\sqrt{ag}$ $\frac{1}{2}m\left(2\sqrt{ag}\right)^2 = \frac{1}{2}mv_C^2 + mga(1+\cos\theta)$	**M1 A1**	**M1** Forms an energy equation from A to C with KE at both points and GPE for a height in terms of θ, using the speed from the impulse. Condone sign slips. **A1** Correct equation, oe; need not be simplified.
	Newton's second law, with acceleration $\frac{v_C^2}{a}$: $T + mg\cos\theta = \frac{mv_C^2}{a},\ T = 0 \Rightarrow v_C^2 = ag\cos\theta$	**M1 A1**	**M1** Resolves towards O at C with $T = 0$, a component of the weight and mass times $\frac{v_C^2}{a}$. Condone sine for cosine and sign slips. **A1** Correct equation with $T = 0$, oe.
	$ag\cos\theta = 2ag(1-\cos\theta) \Rightarrow \cos\theta = \ldots$	**dM1**	Eliminates v_C^2 between their energy and radial equations to reach a value of $\cos\theta$.
	$\cos\theta = \frac{2}{3}$	**A1**	
	$a + a\cos\theta = a + \text{“}\frac{2}{3}\text{”}a = \frac{5a}{3}$	**B1ft**	$\frac{5a}{3}$ oe, or ft $a(1+\text{their}\cos\theta)$.
		[7]	
4(b)	Model ignores air resistance, so no energy is lost Air resistance reduces the speed: $v_C^2 < 2ag(1-\cos\theta)$ $\Rightarrow \cos\theta < \frac{2}{3}$, so the string slackens lower	**E1**	States that air resistance is ignored and that, in reality, it reduces the speed so the string slackens at a lower point. Not for naming the assumption alone.
	$h < \frac{5a}{3}$	**B1**	Condone $\leq$. Accept the inequality in words.
		[2]	
Total for Question 4		**9**	

Question	Scheme	Marks	Notes
5(a)	At B, $T = 0$, with acceleration $\frac{v_B^2}{a}$: $mg\sin\theta = \frac{mv_B^2}{a} \Rightarrow v_B^2 = \ldots$	**M1**	Forms a radial equation at B with zero tension, a resolved weight component and $\frac{mv_B^2}{a}$. Condone sin/cos confusion.
	$v_B^2 = \frac{3ga}{5}$	**A1**	Or $v_B = \sqrt{\frac{3ga}{5}}$, oe.
	B is $\frac{4a}{5} + \frac{3a}{5} = \frac{7a}{5}$ above A	**B1**	
	$\frac{1}{2}mu^2 = \frac{1}{2}m\text{“}\frac{3ga}{5}\text{”} + mg\left(\frac{7a}{5}\right)$ $\left[\Rightarrow u^2 = \frac{17ga}{5}\right]$	**M1 A1**	**M1** Forms an energy equation from A to B with KE at both points and GPE over their height. Condone sign slips. **A1** Correct equation, in v_B or with its value.
	At A, with acceleration $\frac{u^2}{a}$: $T - mg\cos\theta = \frac{mu^2}{a}$	**M1 A1**	**M1** Forms a radial equation at A with tension, a resolved weight component and $\frac{mu^2}{a}$. Condone sign slips and sin/cos confusion. **A1** Correct equation; u^2 need not be substituted.
	$T = \frac{21mg}{5}$	**A1**	oe, such as $4.2mg$.
		[8]	
5(b)	$v_B\cos\theta = \frac{4}{5}v_B$ vertically	**M1**	Resolves their speed at B vertically, or horizontally for an energy method, with the velocity perpendicular to OB. Condone sin/cos confusion.
	$0 = \frac{16}{25} \times \text{“}\frac{3ga}{5}\text{”} - 2gs \Rightarrow s = \ldots$	**dM1**	Uses constant acceleration g vertically, or energy with the horizontal component unchanged, to where the vertical velocity is zero. Condone sign slips.
	$\frac{3a}{5} + \frac{24a}{125} = \frac{99a}{125}$	**A1**	oe, such as $0.792a$. Needs the height of B above O added.
		[3]	
	Total for Question 5	**11**	

Question	Scheme	Marks	Notes
6(a)	$T_A - mg\cos\theta = \frac{mv_A^2}{a}$	**M1 A1**	**M1** Forms an equation of motion towards O at A with tension, a weight component and mass times $\frac{v^2}{a}$. Condone sign slips and sin/cos confusion.
	$T_B + mg\cos\theta = \frac{mv_B^2}{a}$	**M1 A1**	**M1** Forms an equation of motion towards O at B with tension, a weight component and mass times $\frac{v^2}{a}$. Condone sign slips and sin/cos confusion.
	$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mg(2a\cos\theta)$	**M1 A1**	**M1** Forms an energy equation between A and B with both KE terms and a GPE change of the form $kmga\cos\theta$. Condone sign slips.
	$T_A - T_B - 2mg\cos\theta = \frac{m\left(v_A^2 - v_B^2\right)}{a}$ $= \frac{m}{a} \times$ "$4ag\cos\theta$"	**dM1**	Combines their two equations of motion and uses their energy equation to eliminate both speeds.
	$T_A - T_B = 6mg\cos\theta$	**A1 AG**	cso. Correct equations combined without error to the printed result; combined steps accepted.
		[8]	
6(b)	$\frac{1}{2}m(6ag) = \frac{1}{2}mv_B^2 + mga(1+\cos\theta)$	**M1**	Forms an energy equation from the lowest point to B with both KE terms and a GPE change of the form $mga(1 \pm \cos\theta)$. Condone sign slips.
	$v_B^2 = 4ag - 2ag\cos\theta$	**A1**	oe, need not be simplified.
	$5\left(T_B + 6mg\cos\theta\right) = 11T_B \Rightarrow T_B = 5mg\cos\theta$ $\frac{m}{a} \times$ "$4ag - 2ag\cos\theta$" $- mg\cos\theta = 5mg\cos\theta$ $\Rightarrow \cos\theta = \ldots$	**dM1**	Uses $5T_A = 11T_B$ with part (a), or both tensions from equations of motion, and their v_B^2 to reach an equation in $\cos\theta$ only.
	$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$	**A1**	Or $60°$, or awrt 1.05.
	$v_B = \sqrt{3ag}$	**A1**	
		[5]	
	Total for Question 6	**13**	

Question	Scheme	Marks	Notes
7(a)	$\frac{1}{2}m(5ga) = \frac{1}{2}mv^2 + mga(1-\cos\theta)$	**M1**	Forms an energy equation from A to P with KE at both points and the GPE gained. Condone sign slips and sin/cos confusion.
	$v^2 = ga(3+2\cos\theta)$	**A1**	oe
	Towards O, with acceleration $\frac{v^2}{a}$: $R - mg\cos\theta = \frac{mv^2}{a}$	**M1 A1**	**M1** Resolves towards O with R, a weight component and mass times $\frac{v^2}{a}$. Condone sign slips and sin/cos confusion. **A1** Correct equation, in v^2 or with their v^2 substituted.
	$R = 3mg(1+\cos\theta)$	**A1**	oe, given as a positive magnitude.
	"$3mg(1+\cos\theta)$" $= \frac{3}{2}mg \Rightarrow \cos\theta = \ldots$	**M1**	Sets their reaction equal to $\frac{3}{2}mg$ and solves for $\cos\theta$.
	$\cos\theta = -\frac{1}{2}$	**A1**	
	$\theta = \frac{2\pi}{3}$	**A1**	Or $120°$ or awrt 2.09, with no other values in $0 \leq \theta \leq \pi$.
		[8]	
7(b)	$\omega^2 = \frac{v^2}{a^2} = \frac{\text{"}ga(3+2\cos\theta)\text{"}}{a^2}$	**M1**	Uses $v = a\omega$ with their v^2 in terms of θ.
	$\omega = \sqrt{\frac{g(3+2\cos\theta)}{a}}$	**A1**	oe. Not for ω^2 alone.
	Along the tangent, with acceleration $a\frac{\mathrm{d}\omega}{\mathrm{d}t}$: $-mg\sin\theta = ma\frac{\mathrm{d}\omega}{\mathrm{d}t}$	**M1**	Resolves along the tangent with a weight component and mass times $a\frac{\mathrm{d}\omega}{\mathrm{d}t}$, or differentiates their ω^2 with respect to t. Condone sign slips.
	$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{g\sin\theta}{a}$	**A1**	oe, or magnitude $\frac{g\sin\theta}{a}$.
		[4]	
7(c)	$a\omega^2 = g\left(3+2\cos\text{"}\frac{2\pi}{3}\text{"}\right) = \ldots$ $a\frac{\mathrm{d}\omega}{\mathrm{d}t} = -g\sin\text{"}\frac{2\pi}{3}\text{"} = \ldots$	**M1**	Substitutes their θ into both acceleration components, $a\omega^2$ and $a\frac{\mathrm{d}\omega}{\mathrm{d}t}$, from their part (b).
	$2g$ and $-\frac{\sqrt{3}}{2}g$	**A1**	Both components; the sign of the tangential one is not needed.
	$\sqrt{(\text{"}2g\text{"})^2 + \left(\text{"}\frac{\sqrt{3}}{2}g\text{"}\right)^2}$	**M1**	Combines their two components by Pythagoras.

Question 7 continued

Question	Scheme	Marks	Notes
	$\frac{\sqrt{19}}{2}g$	**A1**	Any exact equivalent, such as $\sqrt{\frac{19}{4}}\,g$.
		[4]	
	Total for Question 7	**16**	

Question	Scheme	Marks	Notes
8(a)	$\frac{1}{2}m(10ag) = \frac{1}{2}mv^2 + mg(4a\cos\theta - 2a)$ $\Rightarrow v^2 = 14ag - 8ag\cos\theta$	**M1 A1**	**M1** Forms an energy equation from A to a general position, or to the highest point, with KE at both and a GPE change. Condone sign slips. **A1** Correct equation, oe with the angle from either vertical, or $v^2 = 6ag$ at the top.
	$T + mg\cos\theta = \frac{mv^2}{4a}$	**M1 A1**	**M1** Resolves towards O at a general position, or at the highest point, with T, a weight component and mass times $\frac{v^2}{r}$, where $r = 4a$. Condone sign slips and sin/cos confusion. **A1** Correct equation, or $T + mg = \frac{mv^2}{4a}$ at the top.
	$T + mg\cos\theta = \frac{m(\text{“}14ag - 8ag\cos\theta\text{”})}{4a}$	**dM1**	Substitutes their v^2 from the energy equation to give T in terms of the angle, or at the top.
	$T = \frac{7mg}{2} - 3mg\cos\theta$	**A1**	Or $T = \frac{mg}{2}$ at the top.
	Considers the least tension, at the highest point: $\cos\theta = 1 \Rightarrow T = \frac{7mg}{2} - 3mg = \ldots$	**M1**	Uses $\cos\theta = 1$, or $\cos\theta \leq 1$, in their T, or compares their v^2 at the top with $4ag$, or takes the top as the point of least tension.
	$T = \frac{mg}{2} > 0$, so the string stays taut and the particle performs complete circles	**A1 AG**	Correct equations leading without error to least tension $\frac{mg}{2} > 0$, or $v^2 = 6ag > 4ag$ at the top, with a conclusion that the string stays taut or circles are complete.
		[8]	
8(b)	Lowest point, $\cos\theta = -1$: $T = \frac{7mg}{2} + 3mg = \ldots$	**M1**	Substitutes $\cos\theta = -1$ into their T and compares with $5mg$, or shows their $T = 5mg$ is reached before the lowest point.
	$T = \frac{13mg}{2} > 5mg$, so the string breaks before the lowest point	**A1**	Correct $\frac{13mg}{2}$ compared with $5mg$, or $\theta = 120°$ before the lowest point, with the conclusion that the string breaks first.
	$\frac{7mg}{2} - 3mg\cos\theta = 5mg \Rightarrow \cos\theta = \ldots$	**M1**	Sets their T in terms of the angle equal to $5mg$ and solves for the cosine.
	$\cos\theta = -\frac{1}{2}$ $[\theta = 120°]$	**A1**	
	Angle with the downward vertical $= 60°$	**A1**	Accept $\frac{\pi}{3}$ radians.

Question 8 continued

Question	Scheme	Marks	Notes
	$v^2 = 14ag - 8ag\left(\text{“}-\frac{1}{2}\text{”}\right)$	**M1**	Substitutes their cosine into their v^2 from part (a), or uses energy from A to the breaking point.
	$v = 3\sqrt{2ag}$	**A1**	oe such as $\sqrt{18ag}$.
		[7]	
	Total for Question 8	**15**	

Question	Scheme	Marks	Notes
9(a)	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\left(\frac{3a}{4} - \frac{3a}{2}\cos\theta\right)$ $[v^2 = u^2 - \frac{3ag}{2} + 3ag\cos\theta]$	**M1 A1**	**M1** Forms an energy equation from projection to a general θ, with both KE terms and a GPE change in $\cos\theta$. Condone sign slips. **A1** Correct equation, oe.
	Towards O, with acceleration $\frac{v^2}{r}$, where $r = \frac{3a}{2}$: $T - mg\cos\theta = \frac{2mv^2}{3a}$	**M1 A1**	**M1** Forms a radial equation with T, a weight component and mass times $\frac{v^2}{r}$. Condone sign slips and $\sin\theta$ for $\cos\theta$. **A1** Correct equation, oe.
	$T = mg\cos\theta + \frac{2m}{3a}\left(\text{“}u^2 - \frac{3ag}{2} + 3ag\cos\theta\text{”}\right)$	**dM1**	Substitutes their v^2, in terms of u and θ, into their radial equation.
	$= 3mg\cos\theta - mg + \frac{2mu^2}{3a}$	**A1 AG**	Correct substituted equations leading without error to the printed T; collected steps accepted.
		[6]	
9(b)	Sets $T \geq 0$ at the top, $\cos\theta = -1$: $-3mg - mg + \frac{2mu^2}{3a} \geq 0 \Rightarrow u^2 \geq \ldots$	**M1**	Uses $T \geq 0$ or $T = 0$ at the top of the circle, from the printed T or otherwise, and solves for u or u^2.
	$u = \sqrt{6ag}$	**A1**	oe; $u \geq \sqrt{6ag}$ accepted.
		[2]	
9(c)	Sets $T = 0$ with $u^2 = 4ag$: $3mg\cos\theta - mg + \frac{8mg}{3} = 0 \Rightarrow \cos\theta = \ldots$	**M1**	Sets the printed T to zero with $u^2 = 4ag$ and solves for $\cos\theta$. Condone arithmetic slips.
	$\cos\theta = -\frac{5}{9}$	**A1**	
	$v^2 = 4ag - \frac{3ag}{2} + 3ag\left(\text{“}-\frac{5}{9}\text{”}\right) \Rightarrow v = \ldots$	**dM1**	Uses their $\cos\theta$ in their v^2 from (a), an energy equation, or the radial equation with $T = 0$, to find v.
	$v = \sqrt{\frac{5ag}{6}}$	**A1**	oe, such as $\sqrt{\frac{5}{6}ag}$.
		[4]	
9(d)	Length stays $\frac{3a}{2}$, so P moves on a circle of constant radius $\frac{3a}{2}$, used in the heights and in $\frac{v^2}{r}$	**E1**	States the radius stays $\frac{3a}{2}$ and names at least one use, in the GPE heights or the acceleration $\frac{v^2}{r}$.
		[1]	
	Total for Question 9	**13**	

Question	Scheme	Marks	Notes
10(a)	Component towards O is $\frac{v^2}{L} = 0$ as $v = 0$ at A $\Rightarrow$ no component along OA	**E1**	States that the component towards O is $\frac{v^2}{L}$ (or $L\omega^2$), zero since $v = 0$. Not for "at rest" alone.
		[1]	
10(b)(i)	$\frac{1}{2}m(3gL) = mgL(1 - \cos\theta) \Rightarrow \cos\theta = \ldots$	**M1**	Forms an energy equation from the lowest point with KE there, GPE and $v = 0$ at A, leading to an angle or height. Condone sign slips.
	$\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$	**A1**	Or $60°$ to the upward vertical, or A at height $\frac{3L}{2}$ above the lowest point, oe.
	$m\alpha = mg\sin 60°$	**M1**	Forms Newton's second law along the tangent at A, with the weight component at their angle and no rod force. Condone sin/cos confusion.
	$\alpha = \frac{\sqrt{3}}{2}g$	**A1**	Exact, oe.
		[4]	
10(b)(ii)	Using Newton's second law, with acceleration $\frac{v^2}{L}$: $T + mg\cos 60° = \frac{mv^2}{L} = 0$	**M1**	Resolves along AO with the rod force and the weight component, and sets the acceleration to zero as $v = 0$. Condone sign slips and sin/cos confusion.
	$T = -\frac{mg}{2} \Rightarrow$ magnitude $\frac{mg}{2}$	**A1**	Magnitude $\frac{mg}{2}$ oe.
	Rod in thrust	**A1**	cso, from a correct equation.
		[3]	
10(c)	$h = L - L\cos 60° = \frac{L}{2}$	**B1**	Height above the lowest point, soi in the energy equation.
	$\frac{1}{2}m(3gL) = \frac{1}{2}mv^2 + mg\text{"}\frac{L}{2}\text{"}$	**M1**	Forms an energy equation from the lowest point with both KE terms and GPE, or substitutes $\theta = 60°$ in their general v^2 from (b)(i), which also earns the B1. Condone sign slips.
	$v^2 = 2gL$	**A1**	

Question 10 continued

Question	Scheme	Marks	Notes
	Using Newton's second law, with acceleration $\frac{v^2}{L}$: $T - mg\cos 60° = \frac{mv^2}{L}$	**M1 A1**	**M1** Resolves towards O with tension, the weight component and mass times $\frac{v^2}{L}$. Condone sign slips and sin/cos confusion. **A1** Correct equation; v^2 need not be substituted.
	$T = \frac{5mg}{2}$	**A1**	oe, such as $2.5mg$.
		[6]	
	Total for Question 10	**14**	

Question	Scheme	Marks	Notes
11(a)	$\frac{1}{2}m\left(2\sqrt{ag}\right)^2 = \frac{1}{2}mv^2 + mga$	**M1 A1**	**M1** Forms an energy equation from projection to the highest point with KE at both points and a GPE term. Condone sign slips.
	$v^2 = 2ag$	**A1**	
	Newton's second law, with acceleration $\frac{v^2}{a}$: $T + mg = \frac{mv^2}{a}$	**M1 A1**	**M1** Forms a radial equation with tension, weight and mass times an acceleration of the form $\frac{v^2}{a}$. Condone sign slips. **A1** Correct equation, with v^2 or their value.
	$T = mg$	**A1**	
		[6]	
11(b)	$\frac{1}{2}m(4ag) = \frac{1}{2}mv^2 + mga\sin\theta$ $\Rightarrow v^2 = \ldots$	**M1**	Forms an energy equation to a general position, with KE at both points and GPE in terms of any defined angle. Condone sign slips.
	$v^2 = 4ag - 2ag\sin\theta$	**A1**	oe for their defined angle.
	Newton's second law, with acceleration $\frac{v^2}{a}$: $T + mg\sin\theta = \frac{mv^2}{a}$	**M1 A1**	**M1** Forms a radial equation with tension, a resolved weight component and mass times $\frac{v^2}{a}$. Condone sign slips and sin/cos confusion.
	$T = 4mg - 3mg\sin\theta$	**A1**	oe for their defined angle.
	$4mg - 3mg\sin\theta = 6mg \Rightarrow \sin\theta = \ldots$	**M1**	Sets their tension in terms of the angle equal to $6mg$ and solves for a trigonometric ratio.
	$\sin\theta = -\frac{2}{3}$	**A1**	oe for their defined angle.
	$\theta = 180° + \sin^{-1}\frac{2}{3} \approx 221.8°$	**A1**	awrt 221.8°, as the total angle turned from projection.
		[8]	
11(c)	$v^2 = 4ag - 2ag\left(\text{“}-\frac{2}{3}\text{”}\right)$ $\Rightarrow \frac{v^2}{a} = \ldots$	**M1**	Uses $\frac{v^2}{a}$ with their v^2 from (b), or resolves $T = 6mg$ and the weight along the string, at their breaking position.
	$\frac{v^2}{a} = \frac{16}{3}g$	**A1**	
	$m\alpha = mg\cos\varphi$, $\sin\varphi = \text{“}\frac{2}{3}\text{”}$ $\Rightarrow \alpha = \ldots$	**M1**	Forms a tangential equation of motion with the weight component as the only force, using their angle. Condone sin/cos confusion.

Question 11 continued

Question	Scheme	Marks	Notes
	$\alpha = \frac{\sqrt{5}}{3}g$	**A1**	oe, or awrt $0.745g$.
	$\sqrt{\left(\text{“}\frac{16g}{3}\text{”}\right)^2 + \left(\text{“}\frac{\sqrt{5}g}{3}\text{”}\right)^2} = \ldots$	**M1**	Uses Pythagoras, or a trigonometric ratio for the angle, with their two perpendicular components. Condone the ratio inverted.
	$\sqrt{29}\,g \approx 5.39g$	**A1**	oe, or awrt $5.39g$.
	$\tan\beta = \frac{\sqrt{5}}{16} \Rightarrow \beta \approx 8.0^\circ$	**A1**	awrt 8.0°, measured from the string.
		[7]	
Total for Question 11		**21**	

Question	Scheme	Marks	Notes
12(a)	$h = 2\cos\theta - 1.6$	**B1**	Correct drop below the start, oe such as both heights from any level, $2\cos\theta$ and 1.6 below A.
	$\frac{1}{2} \times 30 \times 2.8^2 + 30g(2\cos\theta - 1.6)$ $= \frac{1}{2} \times 30v^2$	**M1 A1**	**M1** Forms an energy equation with both KE terms and a GPE change in θ. Condone sign slips.
	$117.6 + 60g\cos\theta - 48g = 15v^2$ $\Rightarrow v^2 = 7.84 + 4g(\cos\theta - 0.8)$	**A1 AG**	Correct energy equation leading without error to the printed result; combined steps accepted.
	$0 = 7.84 + 4g(\cos\theta - 0.8) \Rightarrow \cos\theta = \ldots$ $[= 0.6]$	**M1**	Sets $v = 0$ in the printed expression and solves for $\cos\theta$ or θ.
	$\theta = \cos^{-1} 0.6 \approx 53.1°$	**B1**	awrt 53.1°, or awrt 0.927 radians.
		[6]	
12(b)	$v = 2.8$, horizontal $2.8\cos\alpha = 2.24$ vertical $2.8\sin\alpha = 1.68$	**M1**	Resolves their release speed into horizontal and vertical components, with the velocity perpendicular to the rope. Condone sine and cosine swapped.
	$-3.4 = 1.68t - 4.9t^2 \Rightarrow t = \ldots$	**M1**	Uses constant-acceleration vertical motion from release to the ground with their vertical component and their release height, or a trajectory equation for the horizontal distance. Condone sign slips.
	$t \approx 1.02\,\text{s}$	**A1**	awrt 1.02, or awrt 2.29 from the release point by a trajectory equation.
	$1.2 + 2.24 \times$ "1.02" $= \ldots$	**dM1**	Adds their horizontal component times their t to the release point's horizontal distance from O, or adds 1.2 to their horizontal distance from a trajectory equation.
	3.49 m	**A1**	awrt 3.49
		[5]	
12(c)	$T - 30g\cos\theta = \frac{30v^2}{2}$	**M1 A1**	**M1** Resolves towards A, with acceleration $\frac{v^2}{r}$, where $r = 2$, using tension and a weight component. Condone sign slips and sine for cosine.

Question 12 continued

Question	Scheme	Marks	Notes
	$T = 30g\cos\theta + 15\,(7.84 + 4g(\cos\theta - 0.8))$ $= 882\cos\theta - 352.8$	**A1**	oe, such as $90g\cos\theta + 117.6 - 48g$, or a correct tension at the lowest point.
	$\cos\theta = 1 \Rightarrow T = \ldots$ $\cos\theta =$ "0.6" $\Rightarrow T = \ldots$	**M1**	Substitutes $\cos\theta = 1$ and their $\cos\theta$ at rest from (a) into their tension in terms of θ, or resolves at the lowest point and at rest.
	$\cos\theta = 1 \Rightarrow T \approx 529\,\text{N}$ $\cos\theta = 0.6 \Rightarrow T \approx 176\,\text{N}$	**A1**	Both, awrt 529 and 176, identified as greatest and least.
	$529.2 < 600$, so the rope does not break	**A1ft**	ft their greatest tension; compares it with 600 and concludes correctly.
		[6]	
	Total for Question 12	**17**	

Question	Scheme	Marks	Notes
13(a)	$\frac{1}{2}m\left(\frac{gr}{5}\right)+mg\left(\frac{6r}{5}-r\cos\theta\right)$ $=\frac{1}{2}mv^2$	**M1**	Forms an energy equation from A to D with both KE terms and a GPE term for a drop in terms of r and θ. Condone sign slips.
	$v^2=\frac{13gr}{5}-2gr\cos\theta$	**A1**	oe, need not be simplified.
	Newton's second law at D, with acceleration $\frac{v^2}{r}$: $mg\cos\theta-R=\frac{mv^2}{r}$ $R=0\Rightarrow v^2=gr\cos\theta$	**M1 A1**	**M1** Resolves towards O at D with a weight component, acceleration $\frac{v^2}{r}$ and $R=0$, set at any stage. Condone sign slips.
	$gr\cos\theta=$ "$\frac{13gr}{5}-2gr\cos\theta$" $\Rightarrow\cos\theta=\ldots$	**dM1**	Eliminates v^2 between their energy and radial equations and solves for $\cos\theta$.
	$3\cos\theta=\frac{13}{5}\Rightarrow\cos\theta=\frac{13}{15}$	**A1 AG**	Correct energy and radial equations leading without error to $\cos\theta=\frac{13}{15}$; combined steps accepted.
		[6]	
13(b)	$\frac{1}{2}m\left(\frac{gr}{5}\right)+mg\left(\frac{6r}{5}\right)=\frac{1}{2}mV^2$	**M1**	Forms an energy equation to the pool surface from A, or from D with their speed there, with both KE terms and GPE. Condone sign slips.
	$V=\sqrt{\frac{13gr}{5}}$	**A1**	oe, such as $\sqrt{2.6gr}$.
		[2]	
13(c)	Below D	**B1**	Needs a reason seen; not for "below D" alone.
	Lower speed at each point, so $R=mg\cos\theta-\frac{mv^2}{r}>0$ at D	**E1**	States that air resistance lowers the speed, linked to the reaction still positive at D or less force towards O needed. Not for lower speed alone.
		[2]	
13(d)	At C: $R=mg\cos 90^\circ-\frac{mv^2}{r}$ $=-\frac{mv^2}{r}<0$ for any $v>0$	**E1**	States that at C the weight has no component towards O, so R would be negative, or nothing provides the acceleration towards O.
		[1]	
	Total for Question 13	**11**	

Question	Scheme	Marks	Notes
14(a)	$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{1}{2}\sqrt{\gamma r}\right)^2 + m\gamma r(1-\cos\theta)$	**M1 A1**	**M1** Forms an energy equation from A to B with KE at both points and a GPE change in terms of θ. Condone sign slips. **A1** Correct equation, oe.
	$R = 0 \Rightarrow m\gamma\cos\theta = \frac{mv^2}{r}$	**M1 A1**	**M1** Forms an equation of motion towards O at B with a weight component, acceleration $\frac{v^2}{r}$ and $R = 0$. Condone sign slips and sin/cos confusion. **A1** Correct equation, oe such as $v^2 = \gamma r\cos\theta$.
	$\gamma r\cos\theta = \frac{\gamma r}{4} + 2\gamma r(1-\cos\theta)$ $\Rightarrow 3\cos\theta = \frac{9}{4} \Rightarrow \cos\theta = \frac{3}{4}$	**dM1** **A1 AG**	**dM1** Eliminates v^2 between their energy and radial equations to reach an equation in $\cos\theta$ only. **A1 AG** Both equations correct, leading without error to $\cos\theta = \frac{3}{4}$; combined steps accepted.
		[6]	
14(b)	$v^2 = \frac{3}{4}\gamma r$, $\sin\theta = \frac{\sqrt{7}}{4}$ Horizontal: $v\cos\theta = \frac{3\sqrt{3}}{8}\sqrt{\gamma r}$ Vertical: $v\sin\theta = \frac{\sqrt{21}}{8}\sqrt{\gamma r}$	**M1 A1**	**M1** Resolves their speed at B, from their part (a) equations, horizontally and vertically using θ. Condone sin/cos confusion. **A1** Both components correct, in any exact form.
	$\frac{3r}{4} = \text{“}\frac{\sqrt{21}}{8}\sqrt{\gamma r}\text{”}\,t + \frac{1}{2}\gamma t^2$ $\Rightarrow 4\gamma t^2 + \sqrt{21\gamma r}\,t - 6r = 0 \Rightarrow t = \ldots$	**M1**	Uses constant acceleration vertically from B with their height of B above the plane, their vertical component and γ, solving for t, or a trajectory equation for the horizontal distance. Condone sign slips.
	$t = \frac{3\sqrt{13}-\sqrt{21}}{8}\sqrt{\frac{r}{\gamma}}$	**A1**	Positive root only, in any exact form, or the horizontal distance $\frac{9\sqrt{39}-9\sqrt{7}}{64}r$ from B by a trajectory equation.
	$OF = \frac{\sqrt{7}}{4}r + \text{“}\frac{3\sqrt{3}}{8}\sqrt{\gamma r}\text{”} \times \text{“}t\text{”}$	**dM1**	Multiplies their horizontal component by their positive t and adds the horizontal distance $r\sin\theta$ of B from O, or adds $r\sin\theta$ to their horizontal distance from a trajectory equation.
	$OF = \frac{7\sqrt{7}+9\sqrt{39}}{64}r$	**A1**	Any exact form, such as $\frac{\sqrt{7}}{4}r + \frac{9\sqrt{39}-9\sqrt{7}}{64}r$.
		[6]	

Question 14 continued

Question	Scheme	Marks	Notes
14(c)	$m\gamma - R = \frac{mU^2}{r}$, $U^2 > \gamma r \Rightarrow R < 0$ Surface cannot pull, so P leaves at once	**E1**	States that $R < 0$ at A, or that the force needed towards O would exceed the weight, so P cannot stay on the surface.
		[1]	
	Total for Question 14	**13**	

Question	Scheme	Marks	Notes
15(a)	$\cos\theta = \frac{1}{\sqrt{3}}$	**B1**	oe, seen or used.
	$mg\cos\theta = \frac{mv_C^2}{a}$	**M1**	Forms a radial equation at C with zero reaction, the weight component and acceleration $\frac{v^2}{a}$. Condone $\sin\theta$ for $\cos\theta$.
	$v_C^2 = \frac{ag}{\sqrt{3}}$	**A1**	oe, such as $\frac{\sqrt{3}}{3}ag$.
	$\frac{1}{2}mu^2 = \frac{1}{2}mv_C^2 + mga\left(1 + \frac{1}{\sqrt{3}}\right)$	**M1 A1**	**M1** Forms an energy equation from A to C with KE at both points and a GPE change. Condone sign slips and an error in the height. **A1** Correct equation, with the height in any correct form.
	$u^2 = \frac{ag}{\sqrt{3}} + 2ag + \frac{2ag}{\sqrt{3}}$ $= \left(2 + \sqrt{3}\right)ag$	**A1 AG**	Correct v_C^2 substituted into the correct energy equation, leading without error to the printed result; combined steps accepted.
		[6]	
15(b)	$\frac{1}{2}mu^2 = \frac{1}{2}mv_B^2 + mga$	**M1 A1**	**M1** Forms an energy equation between B and A or C with KE at both points and a GPE change. Condone sign slips. **A1** Correct equation, oe from C.
	$v_B^2 = \left(2 + \sqrt{3}\right)ag - 2ag = \sqrt{3}ag$	**A1**	
	$R = \frac{mv_B^2}{a}$	**M1 A1**	**M1** Forms a horizontal equation of motion at B with the reaction as the only force towards O and acceleration $\frac{v^2}{a}$. Not if the weight is included.
	$R = \sqrt{3}mg$	**A1**	Or awrt $1.73mg$.
		[6]	
15(c)	Horizontal $v_C\cos\theta = \frac{v_C}{\sqrt{3}}$ Vertical $v_C\sin\theta = \frac{\sqrt{2}v_C}{\sqrt{3}}$	**M1 A1**	**M1** Resolves the velocity at C, along the tangent, into horizontal and vertical components. Condone sin and cos interchanged. **A1** Both correct, in terms of v_C or its correct value.
	$\frac{v_C}{\sqrt{3}}t = \frac{\sqrt{2}a}{\sqrt{3}} \Rightarrow t = \ldots$	**M1**	Equates their horizontal component times t to a horizontal distance from C to the vertical through O. Condone sin and cos slips.

Question 15 continued

Question	Scheme	Marks	Notes
	$t = \frac{\sqrt{2}\,a}{v_C}$	**A1**	oe, in terms of v_C or of a and g.
	$y = \frac{\sqrt{2}\,v_C}{\sqrt{3}}\,t - \frac{1}{2}gt^2$	**M1**	Uses constant-acceleration vertical motion with their vertical component, their t and acceleration g, or an equivalent trajectory equation. Condone sign slips.
	$y = \frac{2a}{\sqrt{3}} - \frac{ga^2}{\text{“}\frac{ag}{\sqrt{3}}\text{”}}$	**dM1**	Substitutes their v_C^2 from (a), of the form kag, to reach y in terms of a alone. The substitution may be made earlier.
	$y = \frac{2a}{\sqrt{3}} - \sqrt{3}a = -\frac{a}{\sqrt{3}}$ $\frac{a}{\sqrt{3}}$ below C, the level of O, so P passes through O	**A1 AG**	Correct substituted equations leading without error to $y = -\frac{a}{\sqrt{3}}$ oe, with a conclusion that P is then at O.
		[7]	
	Total for Question 15	**19**	