FURTHER MECHANICS / CIRCULAR MOTION & OSCILLATIONS

Motion in a Horizontal Circle

Mark scheme 15 questions · 146 marks

EXAM BOARDS Edexcel FM2 9FM0/4C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Major Y421 CIE 9231 Paper 3

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	Total	**146**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$\omega = \frac{2\pi}{22} = \frac{\pi}{11}\,\text{rad}\,\text{s}^{-1}$	**B1**	Or $v = \frac{2\pi r}{22}$, or awrt 0.286.
	$R\cos\theta = mg$	**M1**	Resolves vertically with the weight and a component of R. Condone sin and cos interchanged.
	$\frac{25}{24}mg\cos\theta = mg \Rightarrow \cos\theta = \frac{24}{25}$	**A1**	Or θ awrt 16.3°, soi.
	$R\sin\theta = mr\omega^2$	**M1 A1**	**M1** Uses Newton's second law towards the centre with a component of R and acceleration $r\omega^2$ or $\frac{v^2}{r}$. Condone sin and cos interchanged. **A1** Correct; values need not be substituted.
	$\frac{25}{24}mg \times \text{“}\frac{7}{25}\text{”} = mr\left(\text{“}\frac{\pi}{11}\text{”}\right)^2$ $\Rightarrow r = \ldots$	**dM1**	Combines their vertical and Newton's second law equations to eliminate θ and solves for r. A numerical θ is accepted.
	$r = \frac{847g}{24\pi^2} \approx 35.0\,\text{m}$	**A1**	
		[7]	
1(b)	The cyclist and bicycle are not a particle: weight and reaction do not act at one point	**E1**	States a real feature the model ignores, such as the size of the cyclist and bicycle or air resistance. Not for “not a particle” alone.
		[1]	
1(c)	Friction, acting up the line of greatest slope	**E1**	States friction acting up the slope; “sideways up the slope” is accepted. Not for friction without this direction.
	The speed is lower, so the particle would tend to slide down the slope; friction opposes this	**E1**	States that the lower speed gives a tendency to slide down the slope, or needs less force towards the centre than the reaction supplies. Not for “friction opposes motion” alone.
		[2]	
Total for Question 1		**10**	

Question	Scheme	Marks	Notes
2(a)(i)	$\frac{3}{5}T + T = 0.8g$	**M1 A1**	**M1** Resolves vertically with a component of T, the reaction equal to T, and the weight. Condone sine and cosine swapped and sign slips.
	$T = 4.9\,\text{N}$	**A1**	
		[3]	
2(a)(ii)	$\frac{4}{5}T = \frac{0.8v^2}{0.4}$	**M1 A1**	**M1** Uses Newton's second law towards O, with a component of T and acceleration $\frac{v^2}{r}$ or $r\omega^2$. Condone sine and cosine swapped and slips in the radius. **A1** Correct, oe in ω, in T or with $T = 4.9$.
	$v = 1.4\,\text{m}\,\text{s}^{-1}$	**A1**	
		[3]	
2(b)	Sets $R = 0$: $\frac{3}{5}T = 0.8g \Rightarrow T = \ldots$	**M1**	Resolves vertically with $R = 0$ towards $T = \ldots$, or with $T = 15$ towards $R = \ldots$. Condone sine and cosine swapped and sign slips.
	$T = \frac{196}{15} \approx 13.1\,\text{N}$	**A1**	Or $R = -1.16$ at $T = 15$, oe.
	$13.1 < 15$, so P loses contact before the string breaks	**A1**	Correct comparison, of tensions or equivalently speeds or R at $T = 15$, and a conclusion that contact is lost first. Need not argue that T increases with speed.
	$\frac{4}{5} \times$ "$\frac{196}{15}$" $= \frac{0.8v^2}{0.4} \Rightarrow v = \ldots$	**M1**	Uses Newton's second law towards O, with a component of their T for $R = 0$, or T eliminated. Condone sine and cosine swapped and slips in the radius.
	$v = \frac{14\sqrt{6}}{15} \approx 2.29\,\text{m}\,\text{s}^{-1}$	**A1**	
		[5]	
Total for Question 2		**11**	

Question	Scheme	Marks	Notes
3(a)	$h^2 + r^2 = 1.7^2$ $(h + 1.2)^2 + r^2 = (4.2 - 1.7)^2$	**M1**	Uses AR from the string length in Pythagoras for both triangles, with h the depth of the centre below B, or in the cosine rule in triangle ABR.
	$2.4h + 1.44 = 3.36 \Rightarrow h = 0.8$ $r^2 = 1.7^2 - 0.8^2 = 2.25 \Rightarrow r = 1.5$	**A1 AG**	Correct equations leading without error to $r = 1.5$; combined steps accepted. Nothing for working back from $r = 1.5$.
		[2]	
3(b)	$T\left(\frac{4}{5} + \frac{8}{17}\right) = 0.4g$	**M1 A1**	**M1** Forms a vertical equation with the weight and the same tension resolved in both sections. Condone sign slips and sine/cosine confusion. **A1** Correct equation, oe. Trigonometric ratios may be decimals.
	$(T =)\ \frac{17g}{54} \approx 3.09\,\text{N}$	**A1**	
		[3]	
3(c)	$T\left(\frac{3}{5} + \frac{15}{17}\right) = 0.4 \times 1.5\,\omega^2$	**M1 A1**	**M1** Forms Newton's second law towards the centre with the same tension resolved in both sections and acceleration $r\omega^2$ or $\frac{v^2}{r}$. Condone sign slips and sine/cosine confusion. **A1** Correct equation, oe, in ω or v. Trigonometric ratios may be decimals.
	"$\frac{126}{85}$" $\times$ "$\frac{17g}{54}$" $= 0.6\omega^2 \Rightarrow \omega^2 = \ldots$ $\frac{2\pi}{\omega} = \ldots$	**dM1**	Substitutes their value of T from (b), or eliminates T with the vertical equation, to find ω or v, then uses $\frac{2\pi}{\omega}$ or $\frac{2\pi r}{v}$.
	$\frac{6\pi}{\sqrt{7g}} \approx 2.28\,\text{s}$	**A1**	
		[4]	
	Total for Question 3	**9**	

Question	Scheme	Marks	Notes
4(a)	$2\sqrt{2}\,\omega = \frac{\pi}{2}\sqrt{1^2+1^2} \Rightarrow \omega = \ldots$	**M1**	Uses the modulus of the velocity as the speed in speed $= 2\sqrt{2}\,\omega$. Condone slips in the modulus.
	$\omega = \frac{\pi}{4}\,\text{rad}\,\text{s}^{-1}$	**A1**	Or awrt 0.785.
		[2]	
4(b)	$2\mathbf{i} - 2\mathbf{j}$ relative to the centre, moving clockwise	**B1**	Both, stated, on a diagram or soi by a correct expression. Sense judged on their axes.
	$\mathbf{r} = \left(3 + 2\sqrt{2}\cos\left(\text{“}\frac{\pi}{4}\text{”}t + \ldots\right)\right)\mathbf{i}$ $+ \left(3 - 2\sqrt{2}\sin\left(\text{“}\frac{\pi}{4}\text{”}t + \ldots\right)\right)\mathbf{j}$	**M1**	Adds the centre's position vector to a rotating radius of length $2\sqrt{2}$, expanded or not, with cos and sin terms in their ωt, any phase. Condone sign slips.
	$\mathbf{r} = \left(3 + 2\sqrt{2}\cos\left(\frac{\pi t}{4} + \frac{\pi}{4}\right)\right)\mathbf{i}$ $+ \left(3 - 2\sqrt{2}\sin\left(\frac{\pi t}{4} + \frac{\pi}{4}\right)\right)\mathbf{j}$	**A1**	oe, including expanded cos and sin terms, with coefficients exact or to 3 s.f.
		[3]	
4(c)	$\mathbf{v} = -\frac{\sqrt{2}\pi}{2}\sin\left(\frac{\pi t}{4} + \frac{\pi}{4}\right)\mathbf{i}$ $- \frac{\sqrt{2}\pi}{2}\cos\left(\frac{\pi t}{4} + \frac{\pi}{4}\right)\mathbf{j}$	**M1 A1**	**M1** Differentiates their trigonometric **r**, a cos or sin term giving the other function with a chain-rule factor, or uses ω^2 times the vector towards the centre. Condone sign slips. **A1** oe. On a direct route to **a**, earned with the final mark.
	$\mathbf{a} = -\frac{\sqrt{2}\pi^2}{8}\cos\left(\frac{\pi t}{4} + \frac{\pi}{4}\right)\mathbf{i}$ $+ \frac{\sqrt{2}\pi^2}{8}\sin\left(\frac{\pi t}{4} + \frac{\pi}{4}\right)\mathbf{j}$	**A1**	oe in t, coefficient awrt 1.74.
		[3]	
4(d)	$\lvert\mathbf{a}\rvert = \frac{\sqrt{2}\pi^2}{8}\,\text{m}\,\text{s}^{-2}$	**B1ft**	ft their **a** with equal amplitudes in its cos and sin terms, or $2\sqrt{2}$ times their ω^2. Correct value exact or awrt 1.74.
		[1]	
4(e)	Takes the point beyond the centre, $5\mathbf{i} + 5\mathbf{j}$: “$3 + 2\sqrt{2}\cos\left(\frac{\pi t}{4} + \frac{\pi}{4}\right)$” $= 5$ “$3 - 2\sqrt{2}\sin\left(\frac{\pi t}{4} + \frac{\pi}{4}\right)$” $= 5 \Rightarrow t = \ldots$	**M1**	Uses the particle at $5\mathbf{i} + 5\mathbf{j}$, or **a** parallel to **r**, to find t from their expressions or from the angle turned and their ω. Not for $t = 2$ or the point $\mathbf{i} + \mathbf{j}$ alone.

Question 4 continued

Question	Scheme	Marks	Notes
	$t = 6$	**A1**	$t = 6$ only.
		[2]	
	Total for Question 4	**11**	

Question	Scheme	Marks	Notes
5(a)	$AR = x,\ BR = 7l - x$ $x^2 + (7l - x)^2 = (5l)^2$	**M1**	Uses Pythagoras or trigonometry in triangle ARB, with the two sections summing to $7l$. Condone sign slips.
	$x^2 - 7lx + 12l^2 = 0 \Rightarrow (x - 3l)(x - 4l) = 0$ $AR = 3l$ or $AR = 4l$	**A1 AG**	Correct substituted equation, solved with working shown to both printed lengths without error; combined steps accepted. Substituting the printed lengths back earns nothing.
		[2]	
5(b)	$\angle RAB = \alpha,\ \angle RBA = \beta$ $T\cos\alpha = T\cos\beta + mg$	**M1**	Resolves vertically for the ring, with the weight and components of both sections in one tension T. Condone sign slips and sine for cosine.
	$\frac{T\times AR}{5l} = \frac{T\times BR}{5l} + mg$	**A1**	Correct equation in the lengths oe, or with the lengths of one case substituted.
	Uses $T > 0$: $T(AR - BR) = 5mgl \Rightarrow AR > BR$	**dM1**	Uses positive tension, or the upward component exceeding the downward, to compare AR and BR, or shows $AR = 3l$ gives $T < 0$.
	$AR = 4l$	**A1**	Printed result, with $AR = 3l$ rejected, stated or implied by $AR > BR$, and no incorrect working.
	$T(4l - 3l) = 5mgl \Rightarrow T = 5mg$	**A1**	Follows the first M1; may use the printed $AR = 4l$.
		[5]	
5(c)	$r = CR = 4l \times \frac{3}{5} = \frac{12l}{5}$	**B1**	
	$T\sin\alpha + T\sin\beta = m \times$ "$\frac{12l}{5}$" $\times \omega^2$	**M1**	Uses Newton's second law towards C with components from both sections and acceleration $r\omega^2$ or $\frac{v^2}{r}$. Condone sign slips and sine for cosine.
	$T\left(\frac{3}{5} + \frac{4}{5}\right) = \frac{12ml\omega^2}{5}$	**A1**	Correct equation with T or $5mg$ and values for the sines, oe in v or the time.
	"$5mg$" $\times \frac{7}{5} = \frac{12ml\omega^2}{5}$ $\omega^2 = \ldots\ [= \frac{35g}{12l}]$ $\frac{2\pi}{\omega} = \ldots$	**dM1**	Substitutes their T of the form kmg, and reaches a time by $\frac{2\pi}{\omega}$, $\frac{2\pi r}{v}$ or equivalent. Condone slips in rearranging.

Question 5 continued

Question	Scheme	Marks	Notes
	$\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{12l}{35g}}$	**A1**	oe, or awrt $3.68\sqrt{\frac{l}{g}}$.
		[5]	
	Total for Question 5	**12**	

Question	Scheme	Marks	Notes
6(a)	$AC = x, \quad DB = y$ $T\sin\alpha = 3m(x\sin\alpha)\omega^2$	**M1**	Forms Newton's second law towards the centre for either particle, with tension resolved (soi) and acceleration $r\omega^2$ or $\frac{v^2}{r}$. Condone slips in the radius or angular speed.
	$T\cos\beta = 2m(y\cos\beta)(2\omega)^2$	**A1**	Both equations correct with the same T, oe such as $T = 3mx\omega^2$ and $T = 8my\omega^2$.
	Equates tensions: "$3mx\omega^2$" = "$8my\omega^2$" $x + y = 11a \Rightarrow x = \ldots$	**dM1**	Eliminates T and ω from their equations for both particles and solves with a total-length equation for x or y. Condone a slip in the total.
	$AC = 8a, \quad DB = 3a$	**A1**	
		[4]	
6(b)	$T\cos\alpha = 3mg \Rightarrow T = \ldots \ [= 5mg]$ $5mg = 3m$ "$8a$" $\omega^2 \Rightarrow \omega = \ldots$	**M1**	Resolves vertically at A and combines with their Newton's second law equation at A or B, with their length, to reach ω or ω^2. Condone $\sin\alpha$ for $\cos\alpha$.
	$\omega = \sqrt{\frac{5g}{24a}}$	**A1**	oe
	"$5mg$" $\sin\beta = 2mg \Rightarrow \sin\beta = \ldots$ $DB\sin\beta =$ "$3a$" $\sin\beta = \ldots$	**M1**	Resolves vertically at B and uses the result with their T and DB, or their ω, to reach the vertical distance. Condone $\cos\beta$ for $\sin\beta$.
	$DB\sin\beta = \frac{6a}{5}$	**A1**	oe
		[4]	
	Total for Question 6	**8**	

Question	Scheme	Marks	Notes
7	$\cos\theta = \frac{4}{5}$ or $r = \frac{3a}{5}$	**B1**	With θ between the radius to P and the downward vertical. Either suffices, or the angle itself, soi; an angle to the horizontal oe.
	$R\cos\theta - \mu R\sin\theta = mg$	**M1 A1**	**M1** Resolves vertically with R, mg and limiting friction, resolving R and friction. Condone sign slips and sine and cosine swapped. Also along the normal or tangent at P, with acceleration components. **A1** Correct oe, with friction μR soi; θ may remain.
	$R\sin\theta + \mu R\cos\theta = m \times \frac{6ga}{5} \div$ "$\frac{3a}{5}$"	**M1**	Uses Newton's second law, with acceleration $\frac{v^2}{r}$ for their r in terms of a, and all relevant forces resolved. Condone sign slips and sine and cosine swapped.
	$R\sin\theta + \mu R\cos\theta = 2mg$	**A1**	Correct oe, with values for v and r; θ may remain.
	$\frac{3+4\mu}{4-3\mu} = 2 \Rightarrow \mu = \ldots$	**dM1**	Eliminates R from their two equations, with friction μR, and solves for μ. Condone slips.
	$\mu = \frac{1}{2}$	**A1**	
	Total for Question 7	**7**	

Question	Scheme	Marks	Notes
8(a)	R and T each at $30°$ to the horizontal	**B1**	Both angles, oe such as $60°$ to the vertical; soi from resolving or a diagram.
	$T = 2mg$	**B1**	
	$R\sin\text{“}30°\text{”} = T\sin\text{“}30°\text{”} + mg$	**M1**	Forms a vertical equation for B with mg and R and T both resolved. Condone sign slips and sine and cosine swapped.
	$R = 4mg$	**A1**	
		[4]	
8(b)	$R\cos\text{“}30°\text{”} + T\cos\text{“}30°\text{”} = mr\omega^2$	**M1**	Uses Newton's second law towards the centre with R and T both resolved and acceleration $r\omega^2$ or $\frac{v^2}{r}$. Condone sign slips, sine and cosine swapped and a wrong radius.
	$(4mg + 2mg)\cos 30° = m \times \frac{\sqrt{3}}{2}a\,\omega^2$	**A1**	oe, with correct values of R, T and the radius.
	$\omega = \sqrt{\frac{6g}{a}}$	**A1**	
		[3]	
	Total for Question 8	**7**	

Question	Scheme	Marks	Notes
9(a)	$T = 5mg$ $T\cos\theta = 3mg \Rightarrow 5mg\cos\theta = 3mg$	**M1**	Uses the tension from A's equilibrium in a vertical equation for B, with a component of T. Condone $\sin\theta$ for $\cos\theta$.
	$\cos\theta = \frac{3}{5}$	**A1**	Printed result from correct equations for A and B, without error.
	$\cos\theta = \frac{RC}{RB} \Rightarrow \frac{3}{5} = \frac{x+a}{3a-x}$	**M1**	Forms an equation in x and a from $\cos\theta$ in triangle RCB, using the string length for RB. Condone the ratio inverted and slips in RC.
	$x = \frac{a}{2}$	**A1**	
		[4]	
9(b)	$T\sin\theta = \frac{3mv^2}{r}$	**M1**	Uses Newton's second law towards C for B, with a component of T and acceleration of the form $\frac{v^2}{r}$ or $r\omega^2$. Condone $\cos\theta$ for $\sin\theta$.
	$r = \frac{5a}{2} \times \frac{4}{5} = 2a$ $5mg \times \frac{4}{5} = \frac{3mv^2}{2a}$	**A1**	Correct equation with values for T, $\sin\theta$ and r, oe such as in ω.
	$v = \sqrt{\frac{8ag}{3}}$	**A1**	oe in a and g.
		[3]	
Total for Question 9		**7**	

Question	Scheme	Marks	Notes
10(a)	$r = 1.2$ $\cos\alpha = \frac{5}{13}$, $\sin\alpha = \frac{12}{13}$ (angle PAB) $\cos\beta = \frac{3}{5}$, $\sin\beta = \frac{4}{5}$ (angle PBA)	**B1**	Radius, and a correct ratio or angle for each string, soi.
	$T_A\cos\alpha = T_B\cos\beta + 0.7g$ $\frac{5}{13}T_A = \frac{3}{5}T_B + 0.7g$	**M1 A1**	**M1** Resolves vertically with three terms, both tensions and the weight. Condone sign slips and sine/cosine confusion. **A1** Correct, oe.
	$T_A\sin\alpha + T_B\sin\beta = 0.7r\omega^2$ $\frac{12}{13}T_A + \frac{4}{5}T_B = 0.7\times 1.2\,\omega^2$	**M1 A1**	**M1** Forms Newton's second law towards the centre with both tensions and acceleration $r\omega^2$ or $\frac{v^2}{r}$. Condone sign slips and sine/cosine confusion. **A1** Correct, oe.
	"$1.44T_B + 1.68g$" $+ 0.8T_B = 0.84\,\omega^2$	**dM1**	Eliminates T_A from their two equations, each containing both tensions.
	$2.24T_B = 0.84\left(\omega^2 - 2g\right)$ $T_B = \frac{3}{8}\left(\omega^2 - 2g\right)$	**A1 AG**	Printed result reached without error from correct vertical and horizontal equations; combined or collected steps accepted.
		[7]	
10(b)	$0 < \frac{3}{8}\left(\omega^2 - 2g\right) \leq 15$ $\Rightarrow 2g < \omega^2 \leq 40 + 2g$	**M1**	Uses the printed tension with both bounds, 0 and 15, and rearranges for ω^2 or ω. Condone equalities or slips in the inequality signs.
	$4.43 < \omega \leq 7.72$	**A1**	Both ends, each with the correct inequality sign; two inequalities given together accepted.
		[2]	
	Total for Question 10	**9**	

Question	Scheme	Marks	Notes
11(a)	$r = 12a$ $\cos\alpha = \frac{4}{5}$, $\sin\alpha = \frac{3}{5}$ $\cos\beta = \frac{3}{5}$, $\sin\beta = \frac{4}{5}$	**B1**	Radius and ratios for the angles α and β that AP and BP make with the vertical, oe, soi.
	$\frac{4}{5}T_A + \frac{3}{5}T_B = mg$	**M1 A1**	**M1** Forms a vertical equation with both tensions resolved and mg. Condone sign slips and sine and cosine interchanged. **A1** oe, with the ratios substituted or not.
	$\frac{3}{5}T_A + \frac{4}{5}T_B = \frac{mv^2}{12a}$	**M1 A1**	**M1** Forms Newton's second law towards the centre with both tensions resolved and acceleration $\frac{v^2}{r}$ oe, for their radius. Condone sign slips and sine and cosine interchanged. **A1** oe, with r and the ratios substituted or not.
	$T_A = \ldots \left[= \frac{5m(16ag - v^2)}{28a}\right]$ $T_B = \ldots \left[= \frac{5m(v^2 - 9ag)}{21a}\right]$	**dM1**	Solves their two equations together for each tension in terms of v^2, or for v^2 at a limiting tension. Condone slips.
	Sets each tension positive and at most $\frac{10mg}{7}$: $0 < \text{“}\frac{5m(16ag - v^2)}{28a}\text{”} \leq \frac{10mg}{7}$ $0 < \text{“}\frac{5m(v^2 - 9ag)}{21a}\text{”} \leq \frac{10mg}{7}$	**M1**	Applies $T > 0$ and $T \leq \frac{10mg}{7}$ to their tensions, at least one of each, reaching bounds on v^2. Condone slips and equalities.
	$8ag \leq v^2 < 16ag$ and $9ag < v^2 \leq 15ag$ $\Rightarrow 9ag < v^2 \leq 15ag$	**A1 AG**	Correct equations and inequalities leading without error to the printed range with its signs. Limiting values alone fall short; the T_A bounds need not be seen.
		[8]	
11(b)	Sets the tensions equal: $\text{“}\frac{5m(16ag - v^2)}{28a}\text{”} = \text{“}\frac{5m(v^2 - 9ag)}{21a}\text{”}$ $\Rightarrow v^2 = \ldots$	**M1**	Sets their tension expressions in v^2 equal, or $T_A = T_B$ in their two equations, and solves for v^2. Condone slips.
	$v^2 = 12ag$	**A1**	
	$\frac{2\pi \times 12a}{\sqrt{12ag}} = 4\pi\sqrt{\frac{3a}{g}}$	**A1**	oe, or awrt $21.8\sqrt{\frac{a}{g}}$.
		[3]	
	Total for Question 11	**11**	

Question	Scheme	Marks	Notes
12(a)	$T\cos\alpha = 0.5\times 9.81, \quad T = \frac{100x}{2}$ $\Rightarrow x = \ldots$	**M1**	Resolves vertically, with a component of T and the weight, and uses Hooke's law for T. Condone sin and cos confused.
	$x = \frac{0.0981}{\cos\alpha}$	**A1**	oe, such as $\frac{4.905}{50\cos\alpha}$.
	$r = (2+x)\sin\alpha = \left(2 + \text{“}\frac{0.0981}{\cos\alpha}\text{”}\right)\sin\alpha$	**B1**	Accept $(2+x)\sin\alpha$ oe, with x not yet replaced.
	$T\sin\alpha = \frac{0.5\times 5^2}{r}$	**M1 A1**	**M1** Applies Newton's second law towards the centre, with a component of T and acceleration $\frac{v^2}{r}$ oe. Condone sin and cos confused. **A1** Correct, oe, with r or a correct expression for it.
	$\frac{4.905\sin\alpha}{\cos\alpha} = \frac{12.5}{\text{“}\left(2+\frac{0.0981}{\cos\alpha}\right)\sin\alpha\text{”}}$	**dM1**	Eliminates T and substitutes their r in terms of α, reaching an equation in α alone. Condone slips.
	$4.905\sin^2\alpha\,(2\cos\alpha + 0.0981) = 12.5\cos^2\alpha$ $\Rightarrow 25\cos^2\alpha = 9.81\sin^2\alpha\,(2\cos\alpha + 0.0981)$	**A1 AG**	Correct substituted equation leading without error to the printed result; combined steps accepted.
		[7]	
12(b)	$\alpha = 55.1^\circ$	**B1**	
		[1]	
12(c)	$x = \frac{0.0981}{\cos\text{“}\alpha\text{”}} = \ldots$	**M1**	Uses their α in their extension in terms of α, or in the vertical equation with Hooke's law.
	$x \approx 0.171$	**A1**	
	$EPE = \frac{100\times\text{“}x\text{”}^2}{2\times 2} = \ldots$	**M1**	Substitutes their extension into $\frac{\lambda x^2}{2l}$ oe. Condone slips in substitution if the formula is quoted correctly.
	$EPE \approx 0.734\,\text{J}$	**A1**	awrt 0.734, from unrounded α or x.
		[4]	
	Total for Question 12	**12**	

Question	Scheme	Marks	Notes
13(a)	$\frac{4}{5}R = 5g, \quad \frac{3}{5}R = \frac{5v^2}{r}$ $\Rightarrow v^2 = \ldots$	**M1**	Forms a vertical equation and Newton's second law towards the centre, with R and weight only, and eliminates R, or one equation along the surface. Condone $\frac{3}{5}$, $\frac{4}{5}$ swapped.
	$v^2 = \frac{3gr}{4}$	**A1**	oe, including with $r = \frac{12}{5}$ already used.
	$r = 3 \times \frac{4}{5} = \frac{12}{5} \Rightarrow v = 4.2\,\text{m s}^{-1}$	**A1**	awrt 4.2. No justification that the natural length gives the greatest speed is needed.
		[3]	
13(b)	$T = \frac{16x}{3}$ $\frac{4}{5}R = 5g + \frac{3}{5}T$	**M1 A1**	**M1** Forms a vertical equation with R, weight and T components, or the equivalent along or normal to the surface, using Hooke's law at any stage. Condone sign slips and $\frac{3}{5}$, $\frac{4}{5}$ swapped. **A1** Both correct, oe.
	$r = \frac{4(3+x)}{5}$ $\frac{3}{5}R + \frac{4}{5}T = \frac{5 \times 6^2}{r}$	**M1 A1**	**M1** Uses Newton's second law towards the centre with R and T components and acceleration $\frac{v^2}{r}$, r including the extension, or equivalently along or normal to the surface. Condone sign slips and ratios swapped. **A1** Both correct, oe.
	$\frac{15g}{4} + \frac{5}{4}T = \frac{225}{3+x}$ $\Rightarrow 80x^2 + 681x - 1377 = 0 \Rightarrow x = \ldots$	**dM1**	Obtains a three-term quadratic in x or the length from their two equations, free of R and T, and solves it by any means.
	$x = \frac{27}{16} = 1.6875\,\text{m}$	**A1**	awrt 1.7, with the negative root rejected.
	$T = 9\,\text{N} \Rightarrow \frac{4}{5}R = 5 \times 9.8 + \frac{3}{5} \times 9$ $R = 68\,\text{N}$	**A1**	awrt 68.
		[7]	
	Total for Question 13	**10**	

Question	Scheme	Marks	Notes
14(a)	OP at 2α to the vertical radius $= r\sin 2\alpha$	**B1**	Both, soi by their use in later equations. Radius oe such as $2r\sin\alpha\cos\alpha$.
	$T\cos\alpha + R\cos 2\alpha = mg$	**M1 A1**	**M1** Resolves vertically with the weight and components of T and R. Also along the normal at P, with acceleration components. Condone sign slips and sine/cosine confusion. **A1** oe
	$T\sin\alpha + R\sin 2\alpha = mr\omega^2\sin 2\alpha$	**M1 A1**	**M1** Uses Newton's second law towards the centre, with components of T and R and acceleration $\omega^2\times$ their radius in r and α. Condone sign slips and sine/cosine confusion. **A1** oe
	Eliminates R, with $\sin 2\alpha = 2\sin\alpha\cos\alpha$: $T\cos 2\alpha + 2\cos\alpha\,(mg - T\cos\alpha)$ $= 2mr\omega^2\cos\alpha\cos 2\alpha$	**dM1**	Eliminates R and uses a double or compound angle identity. A tangent equation at P, free of R, earns the method marks to here, and the accuracy marks if correct.
	$-T + 2mg\cos\alpha = 2mr\omega^2\cos\alpha\cos 2\alpha$ $T = 2m\cos\alpha\left(g - r\omega^2\cos 2\alpha\right)$	**A1 AG**	Correct equation in T without R, leading without error to the printed result; combined steps accepted.
		[7]	
14(b)	$2m\cos\alpha\left(g - r\omega^2\cos 2\alpha\right) + 2R\cos\alpha$ $= 2mr\omega^2\cos\alpha \Rightarrow R = \ldots$	**M1**	Substitutes the printed T into one of their equations in T and R, or eliminates T between them, to reach $R = \ldots$ Condone sign slips.
	$R = m\left(2r\omega^2\cos^2\alpha - g\right)$	**A1**	oe such as $mr\omega^2\left(1 + \cos 2\alpha\right) - mg$
	Requires $T > 0$ and $R > 0$: $2m\cos\alpha\left(g - r\omega^2\cos 2\alpha\right) > 0 \Rightarrow \omega^2 < \ldots$ "$m\left(2r\omega^2\cos^2\alpha - g\right)$" $> 0 \Rightarrow \omega^2 > \ldots$	**M1**	Sets T and their R, in ω^2, positive, or zero at the boundaries, to find both bounds on ω^2. Condone inequality slips.
	$\frac{g}{2r\cos^2\alpha} < \omega^2 < \frac{g}{r\cos 2\alpha}$	**A1**	Both bounds oe with strict inequalities, as one range or two inequalities given together.
		[4]	
	Total for Question 14	**11**	

Question	Scheme	Marks	Notes
15(a)	$\frac{\lambda x}{l} + \mu mg = m(l+x)\omega^2$	**M1 A1**	**M1** Uses Newton's second law towards O with Hooke's law tension, limiting friction and circle radius $l + x$. Condone sign slips.
	$\lambda x + \mu mgl = ml^2\omega^2 + mlx\omega^2$ $\Rightarrow x = \frac{ml\left(l\omega^2 - \mu g\right)}{\lambda - ml\omega^2}$	**A1 AG**	Correct equation of motion leading without error to the printed result; combined or collected steps accepted.
		[3]	
15(b)	$\lambda x = ml\left(l\omega^2 - \mu g\right)\left(1 - \frac{ml\omega^2}{\lambda}\right)^{-1}$	**M1**	Forms λx with a factor $\left(1 \pm \frac{ml\omega^2}{\lambda}\right)^{-1}$ and attempts its binomial expansion, at least two terms. Condone sign slips.
	$\left(1 - \frac{ml\omega^2}{\lambda}\right)^{-1} \approx 1 + \frac{ml\omega^2}{\lambda}$	**A1**	May be seen within λx; higher-order terms may be shown.
	$\lambda x \approx ml\left(l\omega^2 - \mu g\right)\left(1 + \frac{ml\omega^2}{\lambda}\right)$ $\approx ml\left(l\omega^2 - \mu g\right)$	**A1 AG**	Correct expansion leading without error to both printed forms, with $\frac{ml\omega^2}{\lambda}$ stated small or negligible for large λ.
		[3]	
15(c)	$T = \frac{\lambda x}{l} \approx m\left(l\omega^2 - \mu g\right)$ Same as $T + \mu mg = ml\omega^2$ for an inextensible string of length l	**E1**	States the approximate tension, $m\left(l\omega^2 - \mu g\right)$ oe, is that of an inextensible string of length l, or a circle of radius l. Not for the comparison without the tension.
		[1]	
15(d)	$400 \times 0.02 \approx 0.25 \times 0.5\left(0.5\omega^2 - 0.4 \times 9.8\right)$ $\Rightarrow \omega \approx \ldots$	**M1**	Substitutes the given values, g and x in the approximation. Condone x not in metres.
	$\omega \approx 11.7$	**A1**	
		[2]	
15(e)	True radius $0.52 > 0.5$, with the same force $\Rightarrow$ smaller ω, so the estimate is too great	**E1**	States the true radius exceeds 0.5, so ω is smaller and the estimate too great; unchanged force may be implied. Accept ω found with radius 0.52 and compared.

Question 15 continued

Question	Scheme	Marks	Notes
	True $\omega < 11.7 < 12$ so the assumption is justifiable	**E1**	States the assumption is justifiable because true ω is below the estimate, which does not exceed 12. Not for comparing the estimate with 12 alone.
		[2]	
	Total for Question 15	**11**	