Questions

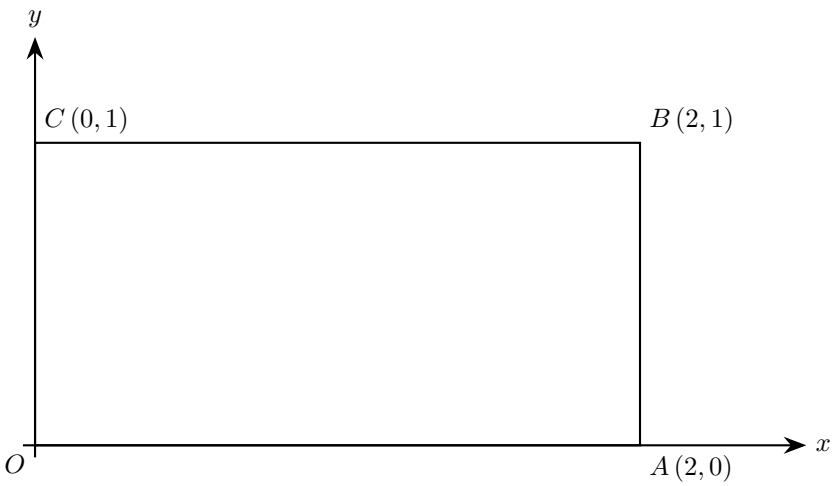
1. Three particles of masses m, $2m$ and km are placed at the points $A(a, 0)$, $B(-2a, 3a)$ and $C(2a, -a)$ respectively, where $k > 0$.

The centre of mass of the three particles is the point with coordinates $(\bar{x}, \bar{y})$.

(a) (i) Find $\bar{x}$ in terms of a and k.
(ii) Find $\bar{y}$ in terms of a and k.

[4]

(b) Given that the centre of mass lies on the circle with centre $(a, 0)$ and radius $\frac{\sqrt{2}}{2}a$, find the possible values of k. **[2]**



2. The diagram shows a light rectangular lamina $OABC$ where O is the origin of the coordinate system in which the points A, B and C have coordinates $(2, 0)$, $(2, 1)$ and $(0, 1)$ respectively. Coordinates refer to the axes shown and the units are metres.

The following three forces are applied to the lamina.

$\begin{pmatrix} p \\ 2q \end{pmatrix}$ N at the point $(1, 0)$

$\begin{pmatrix} -q \\ p \end{pmatrix}$ N at the point C

$\begin{pmatrix} -3 \\ -6 \end{pmatrix}$ N at the point $(t, 1)$, where $0 \leq t \leq 2$

The three forces form a couple of magnitude 3 N m in the anticlockwise direction.

(a) Determine the values of p, q and t. **[5]**

The three forces are removed. The lamina is in the same position as shown in the diagram.

Three particles are now attached to certain points on the lamina as follows.

(b) Find the coordinates of the composite body's centre of mass. Give your answer in terms of λ, where the particles are

- a particle of mass 2 kg at the point $(2\lambda^2, \lambda)$, where $0 \leq \lambda \leq 1$
- a particle of mass 1 kg at the point A
- a particle of mass 1 kg at the point C

[3]

(c) Show that, as λ varies, the centre of mass of the combined object lies on the curve

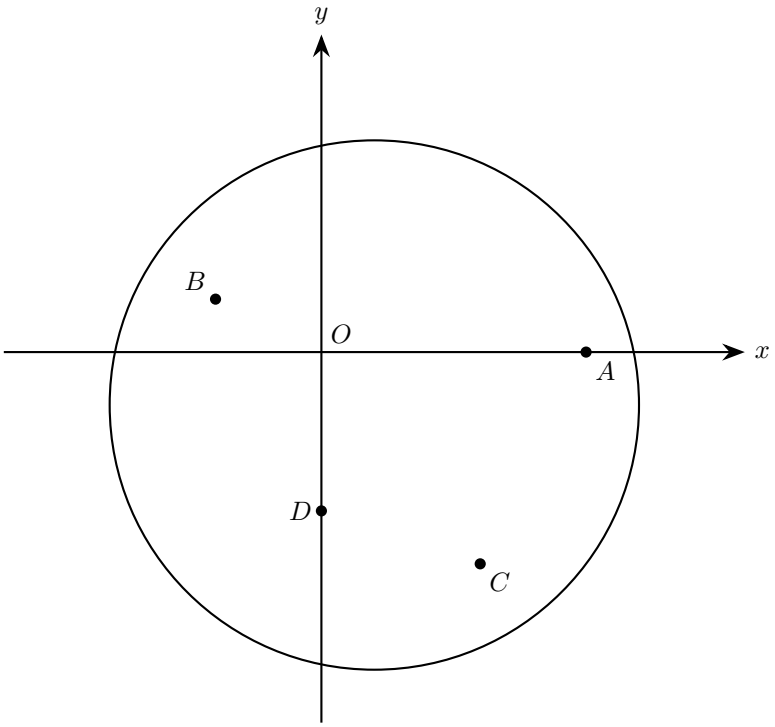
$$x = 4y^2 - 2y + k$$

where k is a constant to be determined. **[3]**

3. Three particles of masses $2m$, $5m$ and $3m$ are placed at the points $(1, -4)$, $(-3, 2)$ and $(q, 2q - 1)$ respectively.

The value of q is such that the distance of the centre of mass of the three particles from the point $(1, 0)$ is as small as possible.

Find the value of q. **[7]**



4. Four sensors are attached to a circular plate.

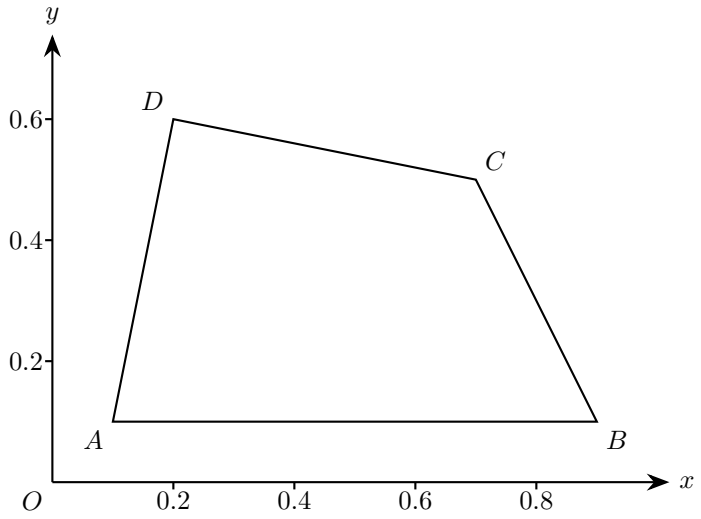
The plate is to be modelled as a uniform lamina and the four sensors as four particles.

The diagram shows the lamina, the four particles A, B, C and D, and the coordinate axes.

The lamina has mass 6 kg and its centre of mass is at the point $(1, -1)$.

Particle A has mass 4 kg and is at the point $(5, 0)$.
Particle B has mass 3 kg and is at the point $(-2, 1)$.
Particle C has mass 5 kg and is at the point $(3, -4)$.
Particle D has mass 2 kg and is at the point $(0, -3)$.

Find the coordinates of the centre of mass of the system of plate and sensors. **[5]**



5. Particles A, B, C and D of masses 1 kg, 2 kg, 3 kg and 4 kg respectively are joined by light rigid rods to form the quadrilateral frame shown in the diagram. The particles are at the points $A(0.1, 0.1)$, $B(0.9, 0.1)$, $C(0.7, 0.5)$ and $D(0.2, 0.6)$, where coordinates are measured in metres.

G, the centre of mass of the frame, has coordinates $(\bar{x}, \bar{y})$.

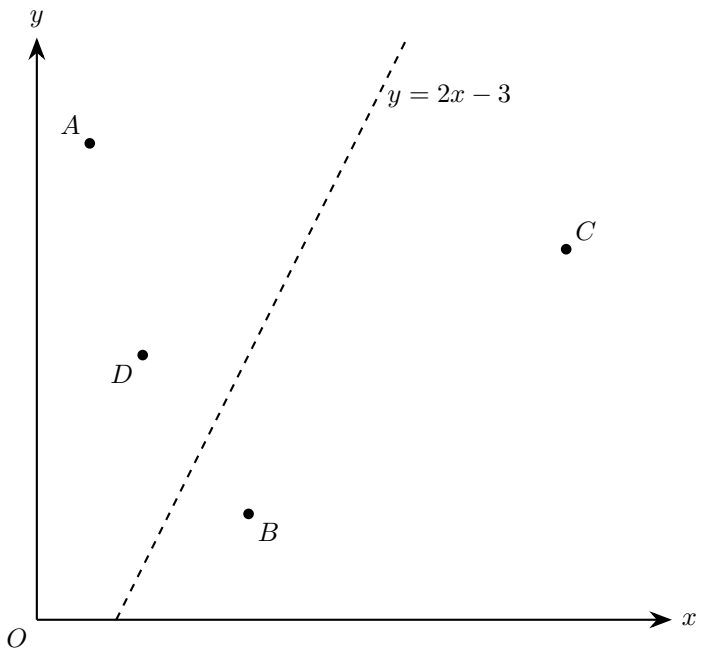
(a) Show that $\bar{y} = 0.42$. Find $\bar{x}$. **[3]**

(b) Using your answer to part (a), explain why the frame cannot be in equilibrium in a horizontal plane if it is supported by a single peg touching the frame at one point. **[1]**

6. Four particles of mass m, $2m$, $4m$ and λm are placed at points with coordinates $(0, 0)$, $(2, 3)$, $(1, -1)$ and $(4, 2)$ respectively. The centre of mass of the system is at $(2, k)$.

(a) Show that $\lambda = 3$. **[4]**

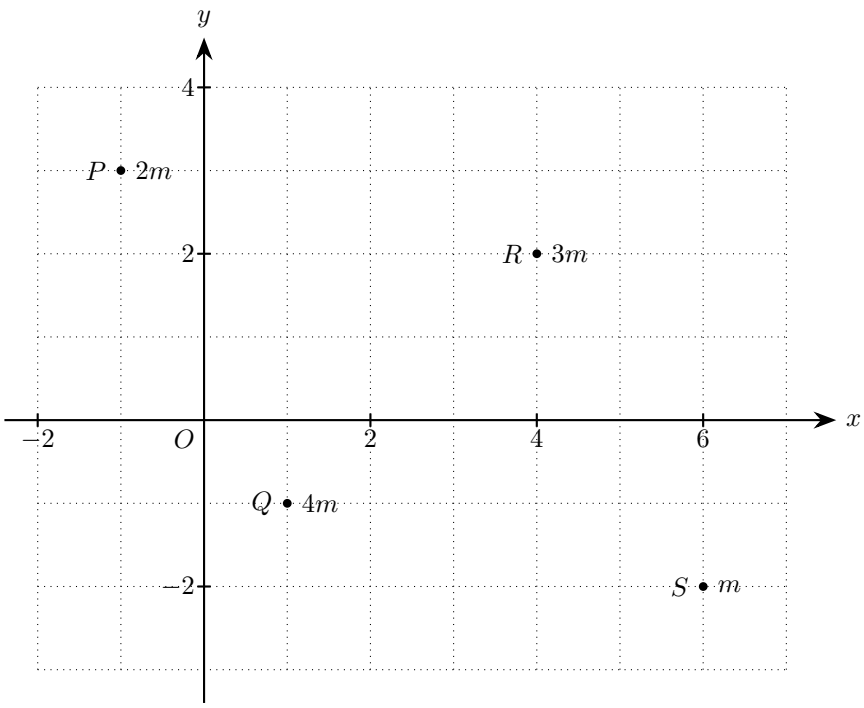
(b) Calculate the value of k. **[3]**



7. The diagram shows four particles, A, B, C and D, fixed in a horizontal plane which contains the x- and y-axes.

Particle A has mass 3 kg and is attached at the point $(1, 9)$.
Particle B has mass 4 kg and is attached at the point $(4, 2)$.
Particle C has mass 5 kg and is attached at the point $(10, 7)$.
Particle D has mass m kg and is attached at the point $(2, 5)$.

The centre of mass of the four particles lies on the line $y = 2x - 3$.
Find the value of m and the coordinates of the centre of mass. **[6]**



8. The diagram shows a system of four particles of masses $2m$, $4m$, $3m$ and m situated in the x-y plane at the points $P(-1, 3)$, $Q(1, -1)$, $R(4, 2)$ and $S(6, -2)$ respectively.

Find the coordinates of the centre of mass of the system. **[5]**

9. Four particles of masses m, $2m$, $3m$ and km are positioned at the points with coordinates $(-a, a)$, $(2a, -2a)$, $(0, 0)$ and $(\mu a, 2\mu a)$ respectively, where k and μ are constants.

The centre of mass of the four particles is at the point with coordinates $(2a, 3a)$.

(a) Find the value of k. **[3]**

(b) Find the value of μ. **[3]**