FURTHER MECHANICS / CENTRE OF MASS

Centre of Mass of Solids of Revolution

Questions by topic 11 questions · 123 marks

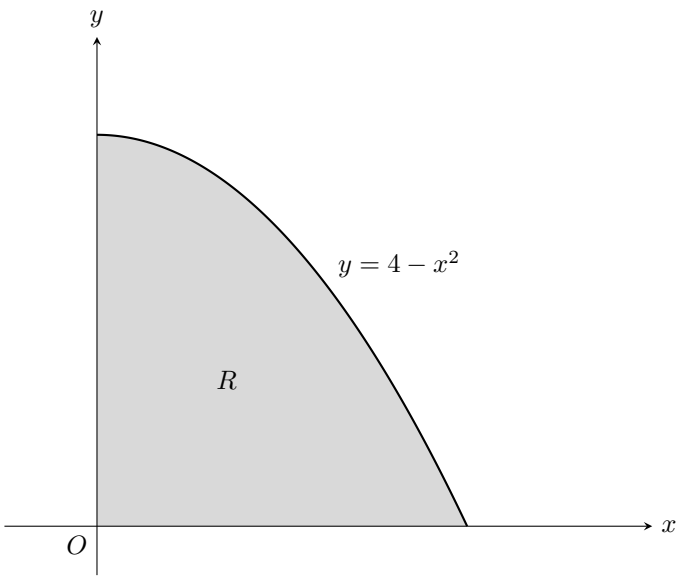
EXAM BOARDS Edexcel FM2 9FM0/4C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Major Y421

Instructions

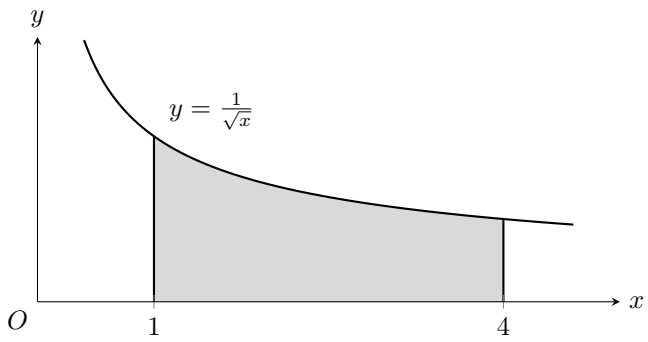
Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question gives a value. Where a question leaves g unspecified and a final answer depends on it, the solutions also give that answer for $g = 10\,\mathrm{m\,s^{-2}}$.

Questions

Question 1	*8 marks*	2
Question 2	*9 marks*	3
Question 3	*12 marks*	4
Question 4	*11 marks*	5
Question 5	*11 marks*	6
Question 6	*14 marks*	7
Question 7	*10 marks*	8
Question 8	*12 marks*	9
Question 9	*12 marks*	10
Question 10	*11 marks*	11
Question 11	*13 marks*	12

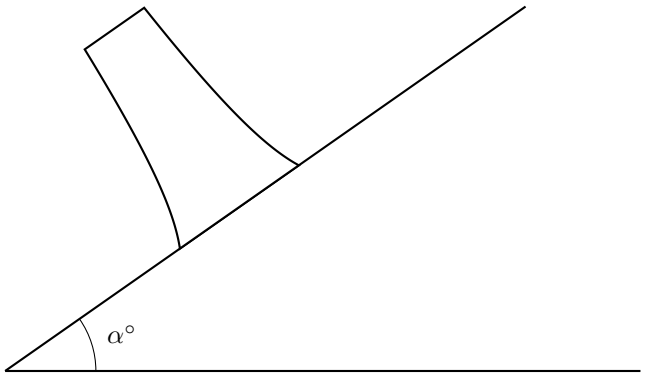


1. The shaded region R, shown in the diagram above, is bounded by the curve with equation $y = 4 - x^2$, the positive x-axis and the positive y-axis. A uniform solid S is formed by rotating R through $360°$ about the x-axis.
Use algebraic integration to find the exact distance of the centre of mass of S from its plane face. **[8]**



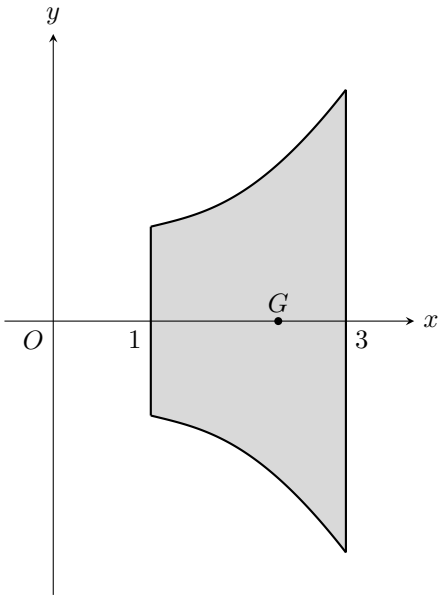
2. The shaded region shown in the diagram above is bounded by the curve with equation $y = \frac{1}{\sqrt{x}}$, the line with equation $x = 1$, the line with equation $x = 4$ and the x-axis.
The region is rotated through 360° about the x-axis to form a solid.
This solid is used to model a uniform solid stand of height 3 m whose plane faces have radii 1 m and 0.5 m.
The larger plane face is the base of the stand.

(a) Show that the centre of mass of the stand is $\left(\frac{3}{2\ln 2} - 1\right)$ m from its base. **[6]**



The stand is placed with its base in contact with a rough plane that is inclined at an angle α° to the horizontal, as shown in the diagram above. The plane is sufficiently rough to prevent the stand from sliding.
Given that the stand is on the point of toppling,

(b) find the value of α, giving your answer to 1 decimal place. **[3]**



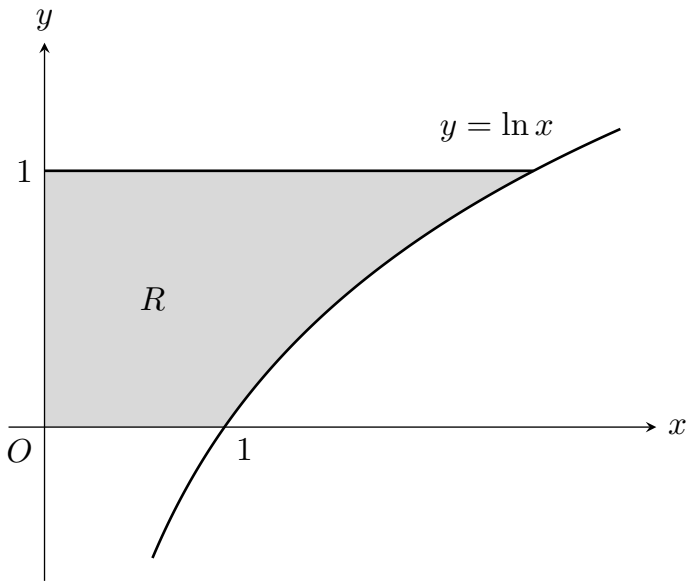
3. The region bounded by the curve $y = \sqrt{x^3 - 3x^2 + 4x}$, the x-axis and the lines $x = 1$ and $x = 3$ is rotated through 2π radians about the x-axis to form a uniform solid of revolution R.
The centre of mass of R is the point G, as shown in the diagram above.

(a) Explain why the y-coordinate of G is 0. **[1]**

(b) Find the x-coordinate of G, giving your answer as an exact fraction. **[8]**

P is a point on the circumference of the smaller plane face of R. The solid R is freely suspended from P and hangs in equilibrium.

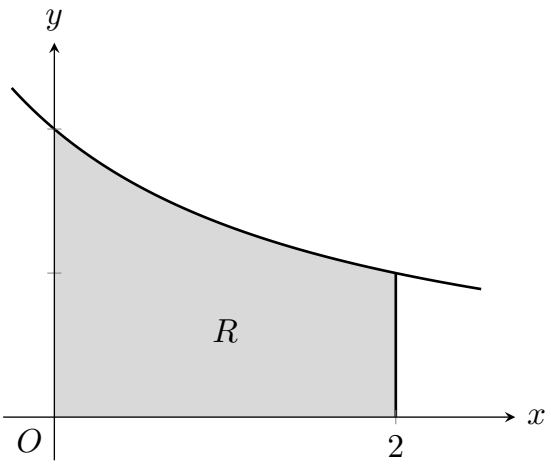
(c) Find the angle between the axis of symmetry of R and the vertical, giving your answer in degrees to 1 decimal place. **[3]**



4. The diagram above shows the region R bounded by the curve with equation $y = \ln x$, the line $y = 1$, the x-axis and the y-axis.
A uniform solid S is formed by rotating R through 2π radians about the y-axis.

(a) Show that the volume of S is $\frac{\pi}{2}\left(e^2 - 1\right)$. **[4]**

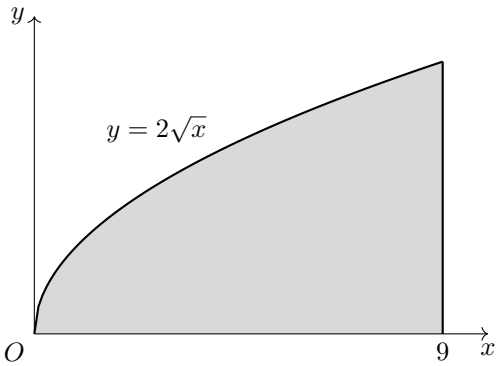
(b) Find, in terms of e, the distance of the centre of mass of S from O. **[7]**



5. The diagram above shows the region R bounded by the curve with equation $y = \frac{4}{2+x}$, the x-axis, the y-axis and the line $x = 2$.
The region R is rotated through 360° about the x-axis to form a uniform solid.

(a) Show that the centre of mass of the solid is at a distance $4(1 - \ln 2)$ from its smaller plane face. **[8]**

(b) The solid is freely suspended from a point on the circumference of its smaller plane face and hangs in equilibrium.
Find the angle between the axis of symmetry of the solid and the vertical, giving your answer to the nearest degree. **[3]**



6. The region bounded by the curve $y = 2\sqrt{x}$, the x-axis and the line $x = 9$, shown in the diagram above, is rotated through 360° about the x-axis to form a solid with a circular plane face.
All units are in centimetres.

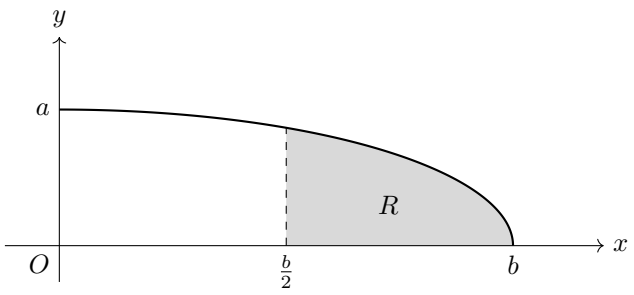
(a) State an assumption that you should make about the solid in order to find the position of its centre of mass. **[1]**

(b) Using integration, show that the centre of mass of the solid is 3 cm from its plane face. **[6]**

(c) The solid is placed with its plane face on a rough board. One end of the board is lifted so that the angle between the board and the horizontal is gradually increased.

(i) Assuming that the solid does not slide, find the angle between the board and the horizontal when the solid is on the point of toppling, giving your answer to the nearest degree. **[3]**

(ii) The coefficient of friction between the solid and the board is 1.5. Determine whether the solid first slides or first topples as the angle is increased, justifying your answer. **[4]**



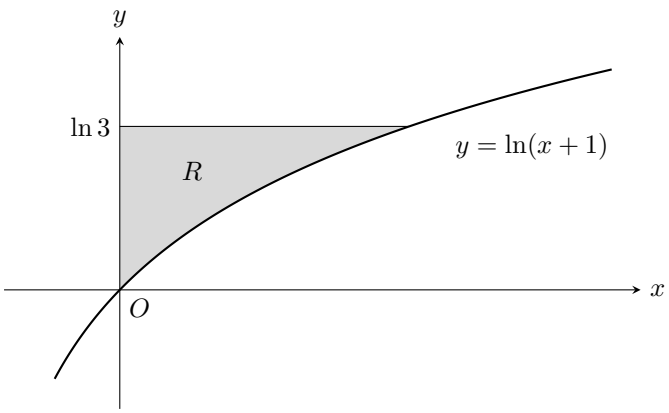
7. The diagram above shows the region R bounded by the curve with equation

$$y = \frac{a}{b}\sqrt{b^2 - x^2}$$

the x-axis and the line $x = \frac{b}{2}$, where a and b are positive constants.
The region R is rotated through 360° about the x-axis to form a uniform solid S.

(a) Use integration to show that the volume of S is $\frac{5}{24}\pi a^2 b$. **[4]**

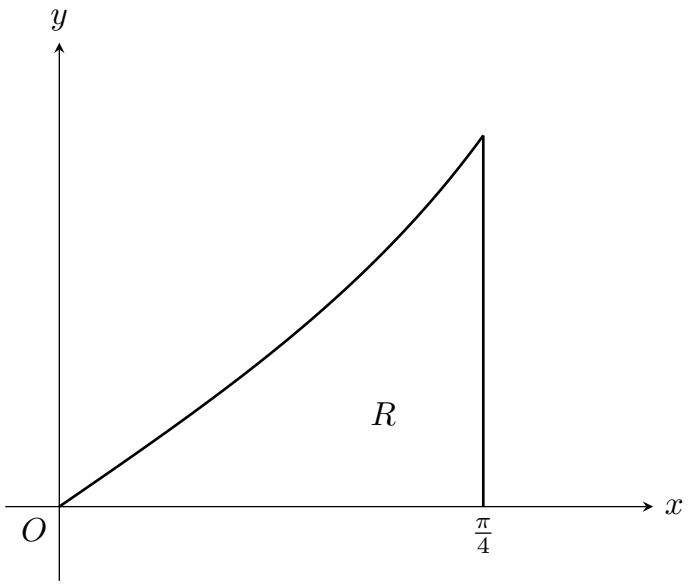
(b) Use integration to show that the centre of mass of S is at a distance $\frac{7b}{40}$ from its plane face. **[6]**



8. The diagram above shows part of the curve with equation $y = \ln(x + 1)$. The region R, shown shaded in the diagram above, is bounded by the curve, the y-axis and the line $y = \ln 3$.
A uniform solid is formed by rotating the region R through 2π radians about the y-axis.

(a) Show that the volume of the solid is $\pi \ln 3$. **[4]**

(b) Find the exact y-coordinate of the centre of mass of the solid. **[8]**

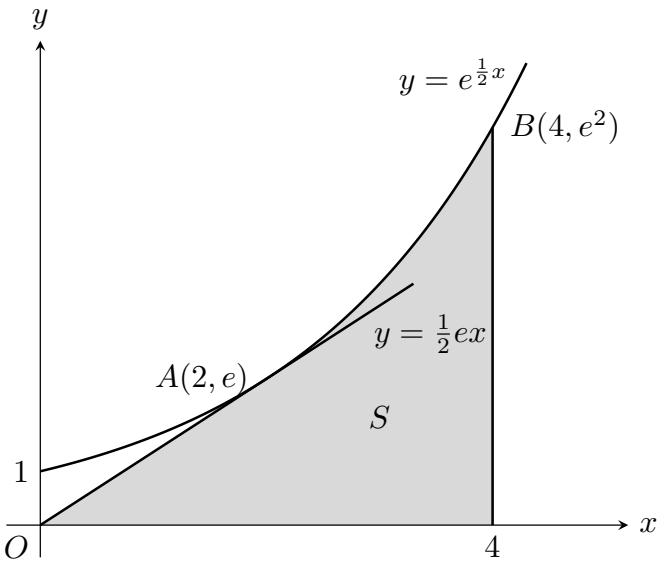


9. The diagram above shows the finite region R which is bounded by part of the curve with equation $y = \tan x$, the x-axis and the line with equation $x = \frac{\pi}{4}$.
A uniform solid S is formed by rotating R through 2π radians about the x-axis.
Using algebraic integration,

(a) show that the volume of S is $\frac{\pi}{4}(4 - \pi)$, **[4]**

(b) find the exact x-coordinate of the centre of mass of S. **[8]**

10. In this question you may use the facts that for a uniform solid right circular cone of height h and base radius r the volume is $\frac{1}{3}\pi r^2 h$ and the centre of mass is $\frac{1}{4}h$ above the base on the line from the centre of the base to the vertex.



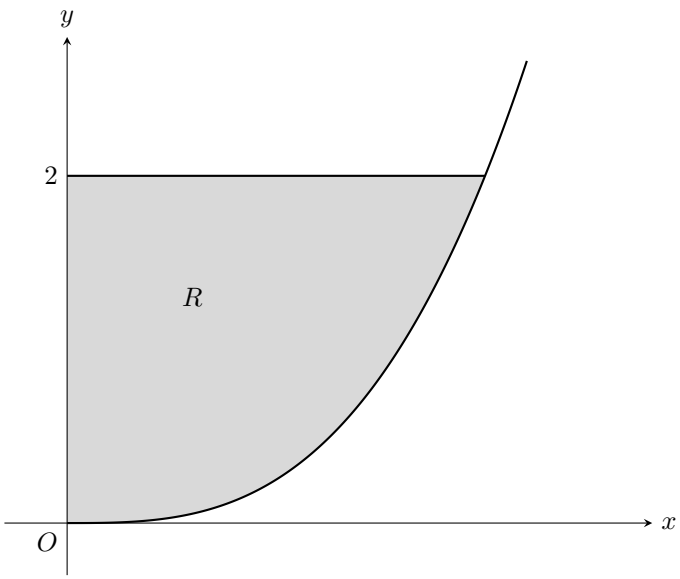
The diagram above shows the shaded region S bounded by the x-axis, the line $y = \frac{1}{2}ex$ for $0 \leq x \leq 2$, the curve $y = e^{\frac{1}{2}x}$ for $2 \leq x \leq 4$, and the line $x = 4$.
The line $y = \frac{1}{2}ex$ is the tangent to the curve $y = e^{\frac{1}{2}x}$ at the point A with coordinates $(2, e)$. The point B on the curve has coordinates $(4, e^2)$.
The region S is rotated through 2π radians about the x-axis to form a uniform solid of revolution T.

(a) Show that the x-coordinate of the centre of mass of T is

$$\frac{9e^2}{3e^2 - 1}$$ **[8]**

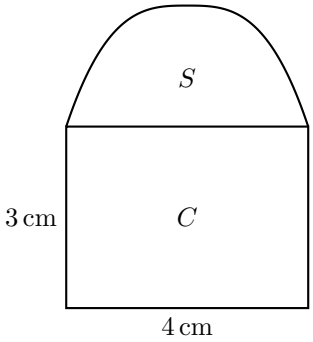
Solid T is freely suspended from B and hangs in equilibrium.

(b) Find the angle between the axis of symmetry of T and the vertical. **[3]**



11. The region R, shown shaded in the diagram above, is bounded by part of the curve with equation $y = \frac{1}{4}x^3$, where $x \geq 0$, the line with equation $y = 2$ and the y-axis. The unit of length on both axes is one centimetre.
A uniform solid, S, is formed by rotating R through 360° about the y-axis.

(a) Find the volume of S, giving your answer as an exact multiple of π, and show that the centre of mass of S is $\frac{3}{4}$ cm from its plane face. **[6]**



A uniform solid cylinder, C, has base radius 2 cm and height 3 cm. The cylinder C is attached to S so that the plane face of S coincides with a plane face of C, forming the paperweight P shown in the diagram above. The density of the material used to make S is k times the density of the material used to make C. The plane face of P rests on a desk lid that is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{4}{5}$. The lid is sufficiently rough to prevent P from slipping.

(b) Find the greatest value of k for which P can rest in equilibrium on the lid without toppling. **[7]**