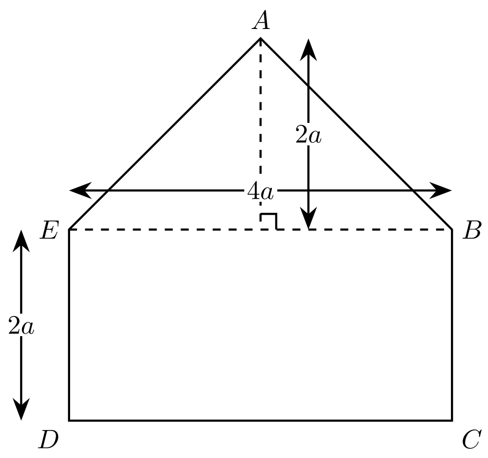
Questions



1. A uniform plane lamina is in the shape of the pentagon $ABCDE$, as shown shaded in the diagram above.

- $BCDE$ is a rectangle
- $BE = 4a$ and $DE = 2a$
- triangle ABE is isosceles with perpendicular height $2a$

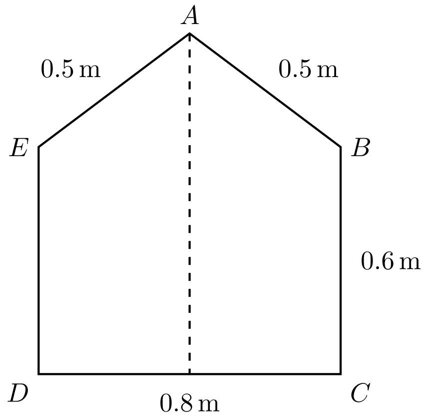
(a) Show that the distance of the centre of mass of the lamina from CD is $\dfrac{14a}{9}$. **[5]**

The mass of the lamina is M.

The lamina is suspended by two light vertical strings, one attached to the lamina at D and the other attached to the lamina at E.

The lamina hangs freely in equilibrium, with DE horizontal.

(b) Using part (a), find, in terms of M and g, the tension in the string attached at E. **[2]**



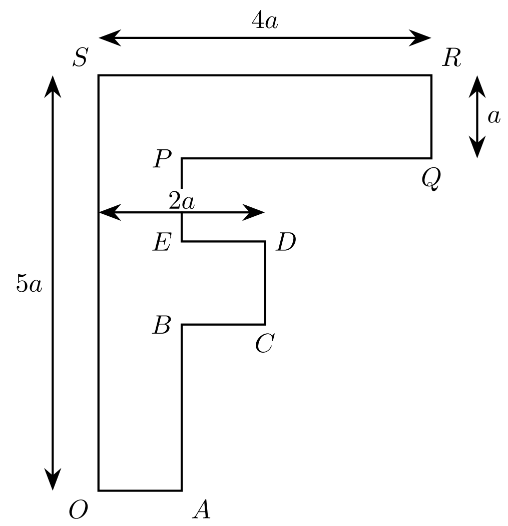
2. $ABCDE$ is a uniform lamina formed by joining an isosceles triangle ABE to the top of a rectangle $BCDE$, where $AB = AE = 0.5\,\mathrm{m}$, $BC = DE = 0.6\,\mathrm{m}$ and $CD = BE = 0.8\,\mathrm{m}$, as shown in the diagram. The centre of mass of the lamina is G.

(a) Explain why G lies on the dashed line. **[1]**

(b) Show that the perpendicular distance of G from CD is $0.38\,\mathrm{m}$. **[4]**

The lamina is freely suspended from the point B.

(c) Determine the acute angle that CD makes with the horizontal, stating which of C or D is higher. **[4]**



3. A letter F from a shop sign is modelled as a uniform plane lamina in the shape shown in the diagram above.

The left-hand vertical strip has width a and height $5a$. The top horizontal strip has length $4a$ and thickness a. The middle horizontal strip has length $2a$ and thickness a.

Using the model,

(a) find, in terms of a, the distance of the centre of mass of the letter F, from

(i) OS

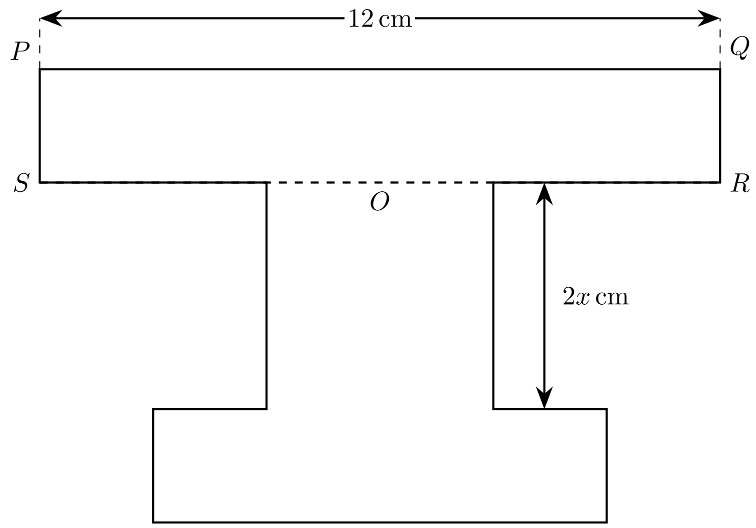
(ii) OA

[6]

The letter F is freely suspended from O and hangs in equilibrium. The angle between OS and the downward vertical is α.

Using the model,

(b) find the exact value of $\tan\alpha$. **[2]**



4. A template T consists of a uniform plane lamina made from three rectangles, as shown in the diagram. The upper rectangle $PQRS$ has $PQ = 12\,\text{cm}$ and $PS = 2\,\text{cm}$. The point O is the midpoint of SR. A rectangle of width $4\,\text{cm}$ and height $2x\,\text{cm}$ is attached centrally below SR, and a rectangle of width $8\,\text{cm}$ and height $2\,\text{cm}$ is attached centrally below this rectangle. The whole template is symmetrical about the line through O perpendicular to SR, where $x > 1$.

(a) Show that the centre of mass of T is a distance

$$\frac{x^2 + 4x - 1}{x + 5}\,\text{cm}$$

below SR. **[7]**

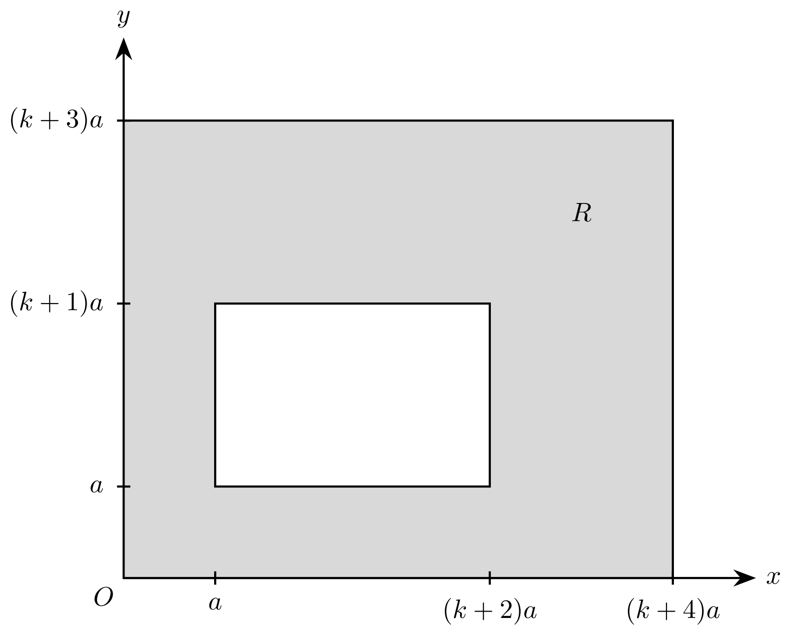
The template T is freely suspended from the point P and hangs in equilibrium.

Given that $x = 2$ and that θ is the angle that PQ makes with the horizontal,

(b) show that

$$\tan\theta = \frac{42}{25}$$

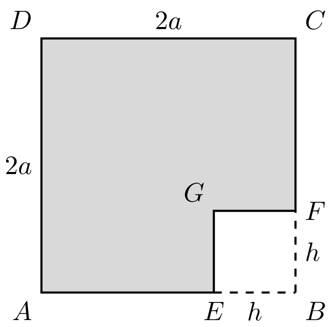
[4]



5. A uniform lamina R is bounded externally by the rectangle with vertices $(0,0)$, $((k+4)a,0)$, $((k+4)a,(k+3)a)$ and $(0,(k+3)a)$, where $k>0$.

A rectangular hole with vertices (a,a), $((k+2)a,a)$, $((k+2)a,(k+1)a)$ and $(a,(k+1)a)$ is removed, as shown in the diagram.

Determine the range of values of k for which the centre of mass of R lies in the hole. **[10]**

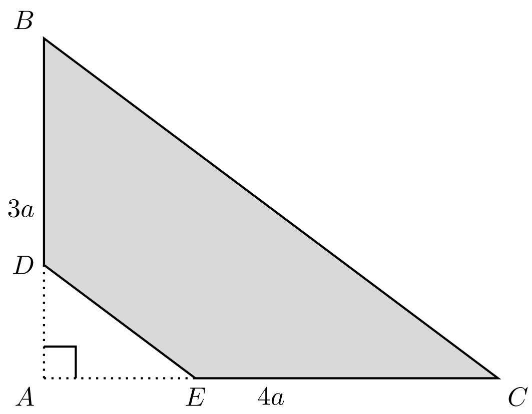


6. A uniform lamina is formed by removing the square $EBGF$ from a square lamina $ABCD$, where $AB = BC = CD = DA = 2a$ and $EB = BF = h$, with $0 < h < 2a$ (see diagram).

(a) Find the distance of the centre of mass of the remaining lamina from AD and from AB, giving your answers in terms of a and h. **[5]**

The lamina is placed vertically on its edge AE on a horizontal plane.

(b) Find, in terms of a, the set of values of h for which the lamina remains in equilibrium. **[3]**

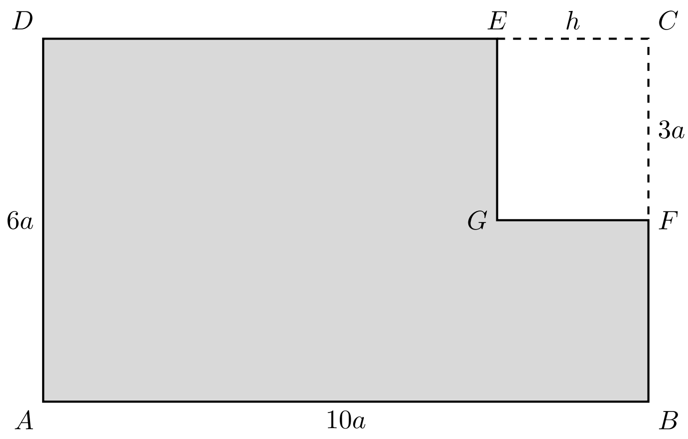


7. A uniform lamina is formed from the right-angled triangle ABC by removing the triangle ADE, as shown in the diagram. The triangle ABC has $AB = 3a$, $AC = 4a$ and $\angle BAC = 90°$. The point D lies on AB and satisfies $AD : AB = 1 : k$. The point E lies on AC, and DE is parallel to BC.

(a) Taking A as the origin, with AC as the x-axis and AB as the y-axis, find the coordinates of the centre of mass of the remaining lamina $DBCE$ in terms of a and k. **[4]**

When the lamina $DBCE$ is freely suspended from C, the edge CE makes an angle θ with the downward vertical, where $\tan\theta = \dfrac{21}{52}$.

(b) Find the value of k. **[3]**



8. $ABCD$ is a uniform rectangular lamina with $AB = 10a$ and $BC = 6a$. Point E lies on DC with $CE = h$, where $0 < h < 10a$, and point F lies on BC with $CF = 3a$. The rectangle $CEGF$ is removed, where G is the fourth vertex of the rectangle (see diagram).

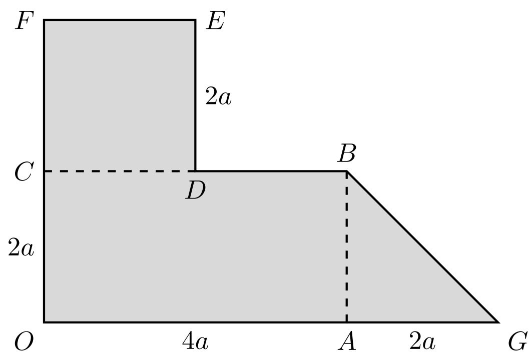
(a) Show that the distance of the centre of mass of the remaining lamina from AD is

$$\frac{h^2 - 20ah + 200a^2}{2(20a - h)}$$

and find a corresponding expression for the distance of the centre of mass from AB. **[5]**

When the lamina is suspended from the point D, the edge DA makes an angle θ with the downward vertical, where $\tan\theta = \frac{13}{12}$.

(b) Find, in terms of a, the two possible values of h. **[3]**



9. The uniform plane lamina shown in the diagram is formed from the rectangle $OABC$, the square $CDEF$ and the triangle ABG. The rectangle has dimensions $4a$ by $2a$, the square has side $2a$ and $AG = 2a$. Take O as the origin, with OA as the positive x-axis and OC as the positive y-axis.

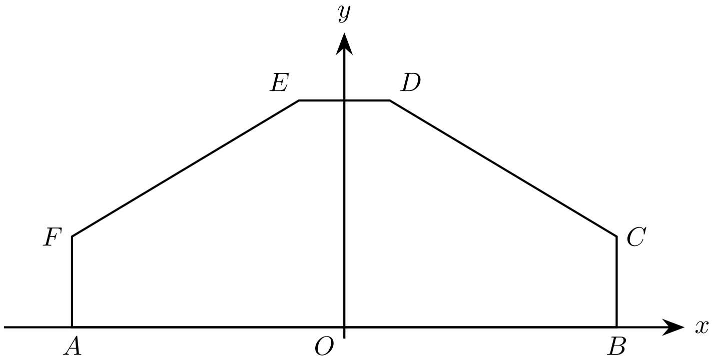
In part (a) you may use, without proof, any of the centre of mass formulae given in the formulae booklet.

(a) Show that the distance of the centre of mass of triangle ABG from AB is $\dfrac{2a}{3}$. **[3]**

(b) Find the coordinates of the centre of mass of the lamina. **[4]**

The lamina is freely suspended from F and hangs in equilibrium with FC at an angle $\theta°$ to the downward vertical.

(c) Find the value of θ. **[4]**



10. A uniform lamina $ABCDEF$ is in the shape of the hexagon shown in the diagram. In a coordinate system with origin O, the vertices have coordinates $A(-120, 0)$, $B(120, 0)$, $C(120, 40)$, $D(20, 100)$, $E(-20, 100)$ and $F(-120, 40)$. The units of the axes are centimetres.

The centre of mass of the lamina lies at $(\bar{x}, \bar{y})$.

(a) Use symmetry to write down $\bar{x}$ and show that $\bar{y} = 40$. **[5]**

The lamina is placed horizontally so that it rests on three supports, whose points of contact are at D, E and G, where G lies on the edge AB and has coordinates $(g, 0)$.

(b) Determine the range of values of g for the lamina to rest in equilibrium. **[3]**

It is now given that $g = 8$, and that the lamina has weight 100N.

(c) Determine the forces exerted on the lamina by each of the supports at D, E and G. **[4]**