

FURTHER MECHANICS / CENTRE OF MASS

Centre of Mass of Circular Laminae

Questions by topic

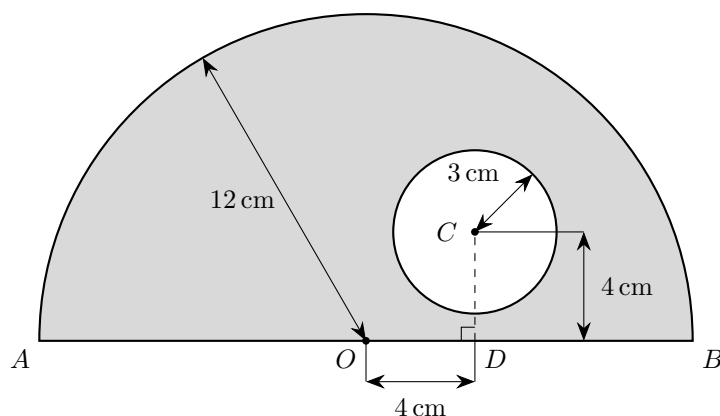
7 questions · 77 marks

Instructions

Unless a question states otherwise, take $g = 9.8 \text{ m s}^{-2}$ wherever a numerical value of g is required. Solutions also give final answers for $g = 10 \text{ m s}^{-2}$ where these differ.

Questions

Question 1	6 marks	2
Question 2	11 marks	3
Question 3	9 marks	4
Question 4	9 marks	5
Question 5	11 marks	6
Question 6	14 marks	7
Question 7	17 marks	8



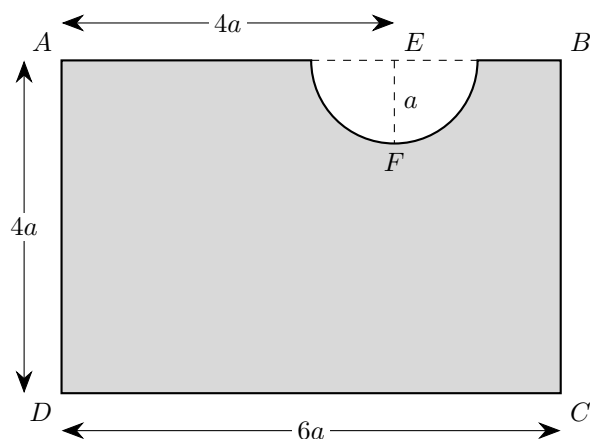
1. A uniform semicircular lamina has radius 12 cm and diameter AB , with midpoint O . A circular hole of radius 3 cm is cut from the lamina. Its centre C is 4 cm above a point D on OB , where $OD = 4$ cm and CD is perpendicular to AB , as shown in the diagram above.

The remaining lamina is freely suspended from B and hangs in equilibrium.

Determine the acute angle that AB makes with the vertical, giving your answer to the nearest 0.1° .

You may use the result that the centre of mass of a uniform semicircular lamina of radius R is on its axis of symmetry at a distance $\frac{4R}{3\pi}$ from its diameter.

[6]



2. A uniform rectangular lamina $ABCD$ has $AB = 6a$ and $AD = 4a$, where $a > 0$.

A semicircular notch of radius a is cut from the edge AB .

The centre E of the semicircle lies on AB , with $AE = 4a$, and the semicircle lies inside the rectangle, as shown in the diagram above.

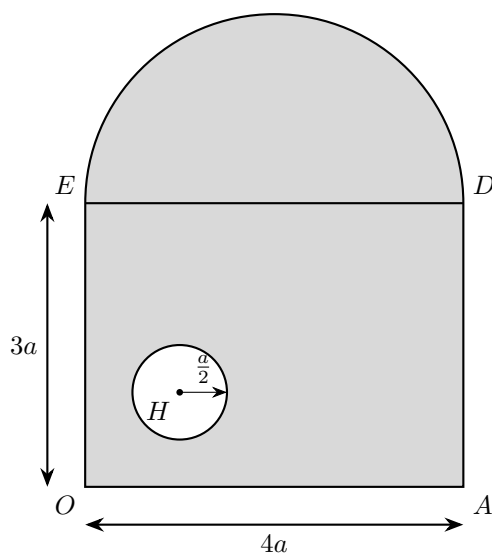
You may use the result that the centre of mass of a uniform semicircular lamina of radius r is a distance $\frac{4r}{3\pi}$ from its diameter.

- (a) (i) Show that the perpendicular distance of the centre of mass of the remaining lamina from AD is

$$\frac{4(36 - \pi)}{48 - \pi}a$$

- (ii) The remaining lamina is freely suspended from A and hangs in equilibrium. Calculate the acute angle that AB makes with the downward vertical, giving your answer to the nearest 0.1° .

[11]



3. A uniform lamina is made by joining a rectangle $OADE$ to a semicircle along its diameter ED , and then removing a circular disc from the rectangular portion, as shown in the diagram above.

The rectangle has $OA = 4a$ and $OE = 3a$, where $a > 0$.

The removed disc has radius $\frac{a}{2}$ and centre H , which is a distance a from each of the sides OE and OA .

You may use the result that the centre of mass of a uniform semicircular lamina of radius r is a distance $\frac{4r}{3\pi}$ from its diameter.

- (a) Find, in terms of a and π , the distance of the centre of mass of the remaining lamina from

- (i) OE ,
- (ii) OA .

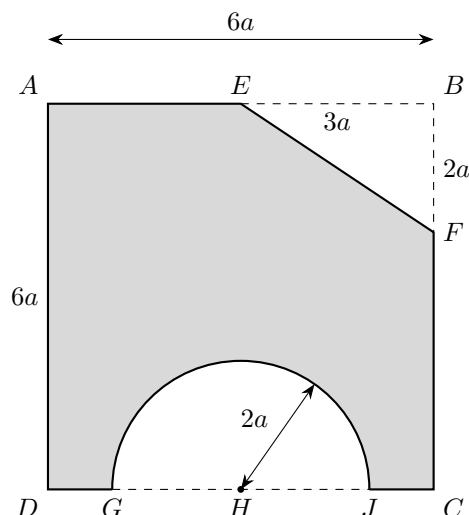
[7]

The remaining lamina is freely suspended from A and hangs in equilibrium.

The angle between AO and the downward vertical is α .

- (b) Find the exact value of $\tan \alpha$.

[2]



4. A uniform square lamina $ABCD$ has side $6a$.

A template T , shown shaded in the diagram above, is made by removing two portions of the lamina.

The first portion is the right-angled triangle EBF , where E lies on AB , F lies on BC , $EB = 3a$ and $BF = 2a$.

The second portion is a semicircle of radius $2a$.

Its centre H is the midpoint of DC , its diameter lies on DC , and the semicircle lies inside the square.

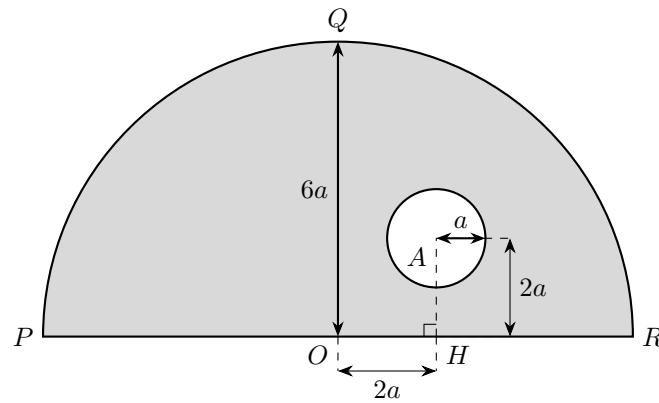
- (a) Find the distances of the centre of mass of T from AD and from DC , giving both answers exactly in terms of a and π .

[6]

The template is now suspended freely from A and hangs in equilibrium in a vertical plane.

- (b) Find the acute angle between AD and the vertical, giving your answer in degrees to one decimal place.

[3]



5. A uniform semicircular lamina has centre O , radius $6a$ and diameter PR .
 A circular disc of radius a is removed to form the lamina L shown shaded in the diagram above.
 The centre of the removed disc is A .
 The point H lies on OR , with $OH = 2a$, $HA = 2a$ and HA perpendicular to PR .

Take O as the origin, with the positive x -axis directed towards R and the positive y -axis directed towards Q .

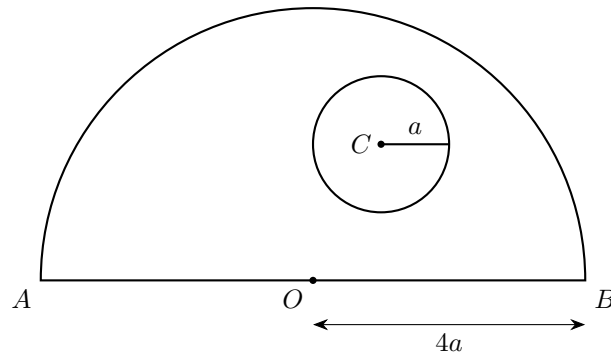
- (a) Find the exact coordinates of the centre of mass of L . [5]

The mass of L is M .

A particle of mass kM is attached at P .

The system is suspended from R and hangs freely in equilibrium, with P below R and the diameter PR making an acute angle of $\arctan\left(\frac{1}{6}\right)$ with the downward vertical through R .

- (b) Find the exact value of k and the magnitude of the supporting force at R in terms of M and g . [6]



6. A uniform semicircular lamina has radius $4a$ and diameter AB , whose midpoint is O .
 A circular hole of radius a is cut from the lamina.
 In the orientation shown above, take O as the origin, the positive x -direction along OB and the positive y -direction perpendicular to AB towards the semicircular arc.
 The centre C of the hole has coordinates $(a, 2a)$.

(i) Show that the centre of mass G of the remaining lamina has coordinates

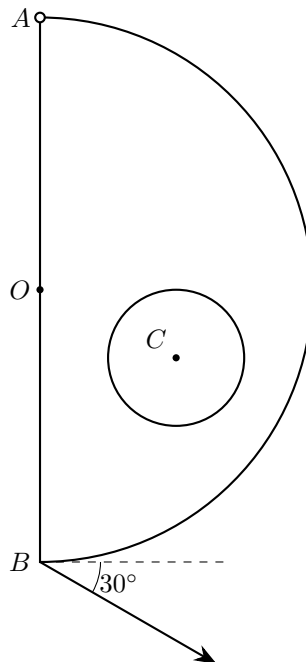
$$\left(-\frac{a}{7}, \frac{(128 - 6\pi)a}{21\pi} \right) \quad [6]$$

The remaining lamina has mass 7 kg .

It is freely pivoted at A and turned so that AB is vertical, with A above B and the semicircular arc to the right of AB , as shown in the diagram below.

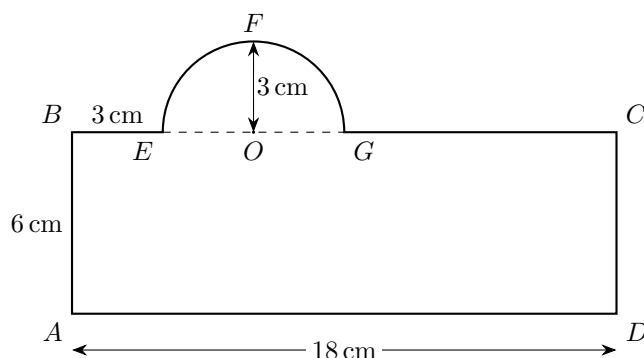
A light string attached to B holds the lamina in equilibrium.

The string lies in the plane of the lamina and makes an angle of 30° below the horizontal.

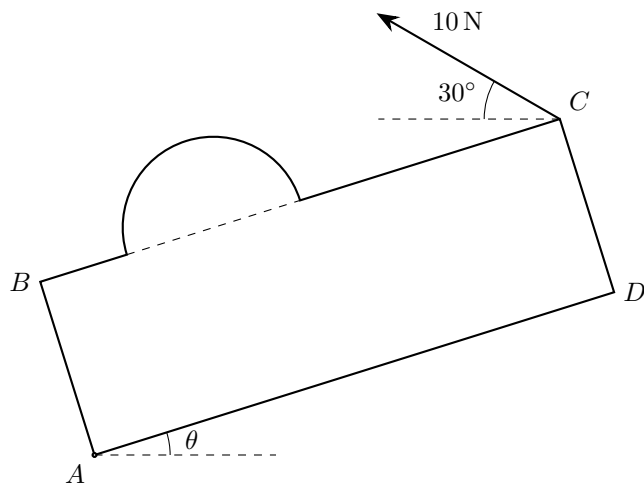


- (ii) Calculate the tension in the string. [3]
- (iii) Find the magnitude of the force exerted on the lamina by the pivot at A and the angle this force makes with the upward vertical, stating on which side of the vertical it acts. [5]

7. A uniform lamina consists of a rectangle $ABCD$ together with a semicircular tab EFG attached externally along its diameter EG , as shown in the diagram below. The two components have the same mass per unit area. The points E and G lie on BC , and O is the midpoint of EG . The dimensions are $AB = 6$ cm, $BC = 18$ cm and $BE = 3$ cm. The semicircular tab has radius 3 cm.



- (i) Consider the semicircular tab EFG on its own. Using the standard centre-of-mass result for a uniform semicircular lamina, show that its centre of mass is 1.27 cm from the diameter EG , correct to 3 significant figures. State the exact distance. [2]
- (ii) For the complete lamina, find
- the distance from AB to its centre of mass, giving your answer exactly, [5]
 - the distance from AD to its centre of mass, giving your answer exactly. [4]
- (iii) The complete lamina has weight 24 N and is smoothly pivoted at A . A wire attached at C supports it in equilibrium. The wire pulls upwards and to the left at 30° above the horizontal, and its tension is 10 N.
- The side AD makes an angle θ above the horizontal, where $0^\circ < \theta < 45^\circ$, as shown in the diagram below.



Calculate θ , giving your answer to 3 significant figures.

[6]