FURTHER MECHANICS / CENTRE OF MASS

Centre of Mass of Composite Solids

Mark scheme 6 questions · 56 marks

EXAM BOARDS Edexcel FM2 9FM0/4C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Minor Y431 OCR MEI Mech Major Y421 · CIE 9231 Paper 3

Questions

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	Total	**56**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	Uniform density and rotational symmetry about OV	**E1**	States both uniformity and symmetry about OV or the axis. Not for symmetry alone.
		[1]	
1(b)	Masses: cylinder $2\pi r^3$, hemisphere $\frac{2}{3}\pi r^3$, cone $\frac{2}{3}\pi r^3$	**B1**	Any proportional masses or volumes, with the hemisphere's counted as negative here or in the moments equation.
	Distances from O: r, $\frac{3}{8}r$, $\frac{5}{2}r$ $[= 0.24, 0.09, 0.6]$	**B1**	All three, measured from O or consistently from another point on the axis.
	$M\bar{x} = \sum m_i x_i$ $\left(2 - \frac{2}{3} + \frac{2}{3}\right) OG$ $= 2r - \frac{2}{3}\left(\frac{3r}{8}\right) + \frac{2}{3}\left(\frac{5r}{2}\right)$ $\Rightarrow OG = \ldots$	**M1**	Forms the moments equation for all three pieces with their masses and distances, the hemisphere subtracted. Need not be simplified.
	$OG = \frac{41r}{24} = 0.41\,\text{m}$	**A1**	
		[4]	
1(c)	Moments about O: $0.48T_A =$ "0.41"$W \Rightarrow T_A = \ldots$	**M1**	Forms a moments equation about any point, each force at its distance from that point, using their OG. Condone sign slips.
	$T_A = \frac{41}{48}W$	**A1**	The first tension found: $T_A = \frac{41}{48}W$ or $T_O = \frac{7}{48}W$; oe, or awrt $0.854W$ or $0.146W$.
	$T_O = W -$ "$\frac{41}{48}W$" $= \frac{7}{48}W$	**A1ft**	The other tension, ft $W-$ their first, oe.
		[3]	
	Total for Question 1	**8**	

Question	Scheme	Marks	Notes
2(a)	Masses $\rho\pi r^3$ and $\frac{16}{3}\rho\pi r^3$ [ratio 3 : 16]	**B1**	Both masses, or any multiples in ratio 3 : 16.
	Heights $\frac{r}{2}$ and $r + \frac{3}{8}(2r) = \frac{7r}{4}$	**B1**	Both heights above O, or from another common reference point used consistently.
	$M\bar{x} = \sum m_i x_i$ $(3+16)\bar{x} = 3\left(\frac{r}{2}\right) + 16\left(\frac{7r}{4}\right)$ $\Rightarrow \bar{x} = \ldots$	**M1**	Uses $M\bar{x} = \sum m_i x_i$ with their masses and heights for both components and M their total mass.
	$\bar{x} = \frac{59}{38}r$	**A1**	
		[4]	
2(b)	Topples when G is above the downhill rim: $\tan\theta = \frac{r}{\text{“}\frac{59}{38}r\text{”}}$	**M1**	Uses $\tan\theta = r \div$ their OG (a multiple of r from (a)), or an equivalent angle from the same triangle. Condone the ratio inverted.
	$\theta = 32.8°$	**A1**	
	$R = Mg\cos\theta,\ F = Mg\sin\theta,\ F = \mu R$	**M1**	Resolves both ways and uses $F = \mu R$. Quoting $\mu = \tan\theta$ at limiting friction earns M1 A1.
	$\Rightarrow \tan\theta = \mu$	**A1**	
	Slides first if it slides at a smaller angle: $\mu <$ tan “32.8°”	**dM1**	Compares μ with the tangent of their toppling angle, by an inequality or by stating which occurs first.
	$0 \leq \mu < \frac{38}{59}$	**A1**	Or awrt 0.644. Condone $\leq$ at the upper end; the lower bound is not required.
		[6]	
	Total for Question 2	**10**	

Question	Scheme	Marks	Notes
3(a)	Cylinder $\rho\pi a^3$, cone $\frac{1}{3}\rho\pi a^2 h$	**B1**	Or in any correct ratio, such as $3a : h$.
	Heights $\frac{a}{2}$ and $a + \frac{h}{4}$ above O	**B1**	Or measured consistently from another point on the axis.
	$\left(\pi a^3 + \frac{1}{3}\pi a^2 h\right)\frac{9a}{10}$ $= \pi a^3 \cdot \frac{a}{2} + \frac{1}{3}\pi a^2 h\left(a + \frac{h}{4}\right)$	**M1 A1**	**M1** Equates total mass times $\frac{9a}{10}$ to the sum of both components' moments, mass times height. Condone slips in the masses or heights. **A1** Correct equation, oe, with or without $\rho\pi$.
	$5h^2 + 2ah - 24a^2 = 0 \Rightarrow (h - 2a)(5h + 12a) = 0$	**dM1**	Rearranges to a three-term quadratic in h and solves it by factorising, the formula or completing the square, leading to $h = \ldots$
	$h = 2a$ ($h = -\frac{12a}{5}$ rejected as $h > 0$)	**A1 AG**	Correct quadratic from a correct equation, solved without error to $h = 2a$; the negative root rejected. Verifying $h = 2a$ by substitution earns neither the dM1 nor this mark.
		[6]	
3(b)	Ornament $\frac{5}{3}\rho\pi a^3$, cavity $\frac{1}{12}\rho\pi a^3$	**B1**	Or in any correct ratio, such as $20 : 1$, or remaining to removed $19 : 1$.
	$\frac{3}{8}\left(\frac{a}{2}\right) = \frac{3a}{16}$	**B1**	
	$\frac{19}{12}\bar{x} = \frac{5}{3} \cdot \frac{9a}{10} - \frac{1}{12} \cdot$ "$\frac{3a}{16}$"	**M1**	Equates the remaining mass times $\bar{x}$ to the ornament's moment minus the cavity's, with $h = 2a$. Condone slips in the masses.
	$\bar{x} = \frac{15a}{16}$	**A1**	oe, or awrt $0.938a$.
		[4]	
3(c)	Centre of mass above the downhill rim: $\tan\theta = \dfrac{a}{\text{"}15a/16\text{"}}$	**M1**	Uses $\tan\theta = \frac{a}{\bar{x}}$, outer radius over height, with their $\bar{x}$ from (b), or an equivalent angle from the same triangle. Condone the ratio inverted.
	$\theta = 47°$	**A1**	
		[2]	
	Total for Question 3	**12**	

Question	Scheme	Marks	Notes
4(a)	Hemisphere : cylinder : cone $\frac{2}{3}\pi r^3 : \pi r^3 : \frac{2}{3}\pi r^3 \quad [= 2:3:2]$	**B1**	Masses in any correct form or ratio; ρ and π may be omitted.
	Distances from O: $-\frac{3r}{8}$, $\frac{r}{2}$, $\frac{3r}{2}$	**B1**	All three, from O or any consistent point on the axis, with the hemisphere on the other side of O.
	$7\bar{x} = 2\left(-\frac{3r}{8}\right) + 3\left(\frac{r}{2}\right) + 2\left(\frac{3r}{2}\right)$	**M1**	Forms total mass times $\bar{x}$ equal to the sum of three mass-distance products, about any point on the axis. Condone sign slips.
	$\bar{x} = \frac{15r}{28}$, on the cylinder and cone side of O	**A1**	Accept awrt $0.536r$. Needs the side, towards the cylinder, cone or A.
		[4]	
4(b)	$(5+h)\bar{x} = 2\left(-\frac{3r}{8}\right) + 3\left(\frac{r}{2}\right)$ $+ h\left(r + \frac{hr}{4}\right)$ $= \frac{r}{4}\left(3 + 4h + h^2\right)$	**M1 A1**	**M1** Forms a moments equation as in (a), with the new cone's mass and distance both in terms of h. Condone slips. **A1** Correct equation, oe and need not be simplified.
	G below B: $\tan 45° = \frac{\bar{x}}{r} \Rightarrow \bar{x} = r$ $(5+h)r = \text{“}\frac{r}{4}\left(3 + 4h + h^2\right)\text{”} \Rightarrow h^2 = \ldots$	**M1**	Uses the centre of mass vertically below B and the right angle at O to obtain $\bar{x} = r$, soi, and substitutes into their equation in h and $\bar{x}$.
	$h = \sqrt{17}$	**A1**	cso. $\sqrt{17}$ only; the negative root rejected or not given.
		[4]	
	Total for Question 4	**8**	

Question	Scheme	Marks	Notes
5(a)	$m_h : m_c = \frac{16}{3}\pi a^3 : \pi a^2 d \ [= 16a : 3d]$	**B1**	Masses or volumes in any correct ratio.
	$\left(0, \frac{3a}{4}\right)$, $\left(\frac{a}{2}, -\frac{d}{2}\right)$	**B1**	Both centres, coordinates possibly stated separately.
	$\left(\frac{16a}{3} + d\right)\bar{x} = d\left(\frac{a}{2}\right)$ $\left(\frac{16a}{3} + d\right)\bar{y} = \frac{16a}{3}\left(\frac{3a}{4}\right) - d\left(\frac{d}{2}\right)$	**M1**	Forms a moment equation for either coordinate: their total mass times it equals the sum of the components' moments. Condone sign slips.
	$\bar{x} = \frac{3ad}{2(16a+3d)}$	**A1**	oe, in terms of a and d.
	$\bar{y} = \frac{24a^2-3d^2}{2(16a+3d)}$	**A1**	oe, in terms of a and d.
		[5]	
5(b)	Vertical through centre of mass meets downhill rim: $\tan\theta = \frac{\frac{3a}{2}-\bar{x}}{d+\bar{y}}$	**M1**	Equates or compares $\tan\theta$ with the centre of mass's distance along the slope from the downhill rim over its height above the plane. Condone sign slips in the offsets.
	$\frac{1}{2} \leq \frac{48a^2+6ad}{3d^2+32ad+24a^2}$	**A1ft**	ft their $\bar{x}$ and $\bar{y}$ from (a), with $\frac{3a}{2}$, d and $\tan\theta = \frac{1}{2}$ correct. Equation or inequality; need not be simplified.
	$3d^2 + 20ad - 72a^2 = 0 \Rightarrow d = \ldots$	**dM1**	Clears fractions to a three-term quadratic in d and solves it by any method.
	$d = \frac{2\sqrt{79}-10}{3}a$	**A1**	Exact, oe. Positive root only, or the negative root rejected.
		[4]	
Total for Question 5		**9**	

Question	Scheme	Marks	Notes
6(a)	Masses $\frac{2}{3}\pi r^3 : \frac{1}{2}\pi r^3 : \frac{1}{3}\pi r^2 h$	**B1**	Masses or volumes in the correct ratio, with any common factor, such as $4r : 3r : 2h$.
	Heights above O: $\frac{5}{8}r$, $\frac{5}{4}r$, $\frac{3}{2}r + \frac{1}{4}h$	**B1**	All three, from O or consistently from another point on the axis.
	$\left(\frac{2}{3}r^3 + \frac{1}{2}r^3 + \frac{1}{3}r^2 h\right) d$ $= \frac{2}{3}r^3 \times \frac{5}{8}r + \frac{1}{2}r^3 \times \frac{5}{4}r$ $+ \frac{1}{3}r^2 h\left(\frac{3}{2}r + \frac{1}{4}h\right)$	**M1 A1**	**M1** Equates total mass times d to the sum of the three mass-height products, with their masses and heights. Condone slips in the masses or heights. **A1** Correct equation, oe, from O or from another stated point. Need not be simplified.
	$d = \dfrac{25r^2 + 12rh + 2h^2}{4(7r + 2h)}$	**A1**	Any equivalent single fraction with integer coefficients, such as denominator $28r + 8h$, measured from O.
		[5]	
6(b)	Centre of mass below centre of hemisphere: $d < r$	**B1**	Stated or used. Condone $d \leq r$ here.
	"$\dfrac{25r^2 + 12rh + 2h^2}{4(7r + 2h)}$" $< r$ $\Rightarrow 2\left(\frac{h}{r}\right)^2 + 4\left(\frac{h}{r}\right) - 3 < 0$	**M1**	Sets their d, a quadratic over a linear expression, less than r, clearing the denominator to a three-term quadratic in $\frac{h}{r}$, or in h and r. Condone $=$ or $\leq$.
	$\left(\frac{h}{r} + 1\right)^2 < \frac{5}{2} \Rightarrow \frac{h}{r} < \ldots$	**dM1**	Finds the critical values of their quadratic by formula, completing the square or calculator, and takes the interval between them.
	$0 < \dfrac{h}{r} < \dfrac{\sqrt{10} - 2}{2}$	**A1**	Strict inequalities, any exact form such as $\sqrt{\frac{5}{2}} - 1$. The lower bound 0 may be omitted; A0 for a negative lower bound.
		[4]	
Total for Question 6		**9**	