FURTHER MECHANICS / CENTRE OF MASS

Centre of Mass of Circular Laminae

Mark scheme 7 questions · 76 marks

EXAM BOARDS Edexcel FM2 9FM0/4C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Minor Y431 OCR MEI Mech Major Y421 · CIE 9231 Paper 3

Questions

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	Total	**76**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1	$\mathcal{A} = 72\pi - 9\pi = 63\pi$	**B1**	Areas in any correct ratio, such as $8:1:7$; may be implied by the moment equations.
	Semicircle: $\frac{4\times12}{3\pi} = \frac{16}{\pi}$ from AB	**B1**	Or awrt 5.09.
	$63\pi\bar{x} = 72\pi \times 0 - 9\pi \times 4 \Rightarrow \bar{x} = \ldots$	**M1**	Takes moments about any axis perpendicular to AB, with all nonzero terms and the hole's moment subtracted. Condone slips in values.
	$\bar{x} = -\frac{4}{7}$	**A1**	Or $\frac{4}{7}$ from O towards A, or the equivalent distance from any other stated axis, such as $\frac{88}{7}$ from B.
	$63\pi\bar{y} = 72\pi \times$ "$\frac{16}{\pi}$" $- 9\pi \times 4$ $\Rightarrow \bar{y} = \ldots$	**M1**	Takes moments about AB or a parallel axis, with their distance of the semicircle's centre of mass from AB and the hole's moment subtracted. Condone slips in values.
	$\bar{y} = \frac{128-4\pi}{7\pi}$ $[= 5.249\ldots]$	**A1**	oe, or awrt 5.25.
	G vertically below B: $\tan\theta = \frac{\bar{y}}{12-\bar{x}} = \frac{32-\pi}{22\pi}$ $[= 0.4175\ldots]$	**M1 A1ft**	**M1** Uses G vertically below B and forms tan of an angle at B from the distances of G along and perpendicular to AB. Condone the ratio inverted and sign slips. **A1ft** ft their $\bar{x}$ and $\bar{y}$, with the distance of G from B along AB in the denominator.
	$\theta = 22.7°$	**A1**	awrt 22.7°.
	Total for Question 1	**9**	

Question	Scheme	Marks	Notes
2(a)(i)	$\mathcal{A} = 24a^2 - \frac{1}{2}\pi a^2 \ \left[= \frac{48-\pi}{2}a^2\right]$	**B1**	Any equivalent form.
	$\mathcal{A}\bar{x} = 24a^2(3a) - \frac{1}{2}\pi a^2(4a)$	**M1**	Takes moments about AD or a parallel axis, with rectangle and semicircle terms, the semicircle's subtracted. Condone one slip in a distance.
	$\frac{48-\pi}{2}a^2\bar{x} = (72-2\pi)a^3$ $\Rightarrow \bar{x} = \frac{4(36-\pi)}{48-\pi}a$	**A1 AG**	Correct substituted moment equation leading without error to the printed result; combined steps accepted.
		[3]	
2(a)(ii)	Semicircle: $4a - \frac{4a}{3\pi}$ above DC	**B1**	Or $\frac{4a}{3\pi}$ below AB, with moments about AB.
	$\mathcal{A}\bar{y} = 24a^2(2a) - \frac{1}{2}\pi a^2\left(\text{“}4a - \frac{4a}{3\pi}\text{”}\right)$	**M1**	Takes moments about DC, AB or a parallel axis, with rectangle and semicircle terms, the semicircle's subtracted, using their semicircle distance. Condone sign slips.
	$\bar{y} = \frac{292-12\pi}{3(48-\pi)}a \ [\approx 1.89a]$	**A1**	Or distance from AB $\frac{284}{3(48-\pi)}a$. Any exact form, or awrt $1.89a$ or $2.11a$.
	Uses G vertically below A: $\tan\theta = \frac{4a-\text{“}\bar{y}\text{”}}{\bar{x}}$	**M1**	Forms tan of the angle between AG and AB or AD from the distances of G from AB and AD. Condone the ratio inverted.
	$= \frac{71}{3(36-\pi)} \ [\approx 0.720]$	**A1ft**	ft their distance of G from AB with the printed $\bar{x}$. Accept the reciprocal for the angle with AD.
	$\theta = 35.8°$	**A1**	awrt $35.8°$.
		[6]	
	Total for Question 2	**9**	

Question	Scheme	Marks	Notes
3(a)	$\mathcal{A} = 12a^2 + 2\pi a^2 - \frac{\pi a^2}{4}$ $\left[= \frac{a^2(48+7\pi)}{4}\right]$	**B1**	All three areas, the disc's subtracted. Need not be simplified.
	From OE: $2a,\ 2a,\ a$ From OA: $\frac{3a}{2},\ 3a + \frac{8a}{3\pi},\ a$	**B1**	All six distances, oe.
	$\mathcal{A}\bar{x} = 12a^2(2a) + 2\pi a^2(2a) - \frac{\pi a^2}{4}(a)$ $\Rightarrow \bar{x} = \ldots$	**M1**	Forms a moment equation about OE or a parallel line for all three pieces, the disc subtracted, with their total area. Condone slips in areas and distances.
	(i) $\bar{x} = \frac{a(96+15\pi)}{48+7\pi}$	**A1**	Any exact equivalent.
	$\mathcal{A}\bar{y} = 12a^2\left(\frac{3a}{2}\right) + 2\pi a^2\left(3a + \frac{8a}{3\pi}\right)$ $- \frac{\pi a^2}{4}(a) \Rightarrow \bar{y} = \ldots$	**M1**	Forms the corresponding moment equation about OA or a parallel line, with the same allowances.
	(ii) $\bar{y} = \frac{a(280+69\pi)}{3(48+7\pi)}$	**A1**	Any exact equivalent.
		[6]	
3(b)	G vertically below A: $\tan\alpha = \frac{\text{“}\bar{y}\text{”}}{4a - \text{“}\bar{x}\text{”}}$	**M1**	Forms $\tan\alpha$ as the distance of G from OA over its distance from AD, using their (a) values in a and π. Not for $\frac{\bar{y}}{\bar{x}}$. Condone the correct ratio inverted.
	$\tan\alpha = \frac{280+69\pi}{3(96+13\pi)}$	**A1**	Any exact equivalent, the right way up, free of a.
		[2]	
	Total for Question 3	**8**	

Question	Scheme	Marks	Notes
4(a)	$\mathcal{A} = 36a^2 - 3a^2 - 2\pi a^2$ $[= (33 - 2\pi)a^2]$	**B1**	All three areas, with both removed pieces subtracted.
	Triangle $\left(5a, \frac{16a}{3}\right)$ Semicircle $\left(3a, \frac{8a}{3\pi}\right)$	**B1**	Both centres of mass, as coordinates from D or equivalent positions relative to any consistent axes.
	$(33 - 2\pi)a^2\bar{x} = 36a^2(3a) - 3a^2(5a)$ $- 2\pi a^2(3a)$	**M1**	Forms a moment equation about AD, or a parallel line, with all three pieces, using their areas and centres of mass. Condone a sign slip.
	$\bar{x} = \frac{(93-6\pi)a}{33-2\pi}$	**A1**	Any exact equivalent, as a distance from AD.
	$(33 - 2\pi)a^2\bar{y} = 36a^2(3a) - 3a^2\left(\frac{16a}{3}\right)$ $- 2\pi a^2\left(\frac{8a}{3\pi}\right)$	**M1**	Forms a moment equation about DC, or a parallel line, with all three pieces, using their areas and centres of mass. Condone a sign slip.
	$\bar{y} = \frac{260a}{3(33-2\pi)}$	**A1**	Any exact equivalent, as a distance from DC.
		[6]	
4(b)	$\tan\theta = \dfrac{\text{“}\bar{x}\text{”}}{6a - \text{“}\bar{y}\text{”}}$	**M1**	Uses the centre of mass vertically below A and forms $\tan\theta$ from its offsets from A, their $\bar{x}$ and $6a - \bar{y}$. Condone the ratio inverted.
	$\tan\theta = \frac{279-18\pi}{334-36\pi}$ $[\approx 1.007]$	**A1ft**	ft their $\bar{x}$ and $\bar{y}$ from (a), the right way up; exact, or awrt 1.01 from correct values.
	$\theta = 45.2°$	**A1**	awrt $45.2°$.
		[3]	
Total for Question 4		**9**	

Question	Scheme	Marks	Notes
5(a)	$\mathcal{A}$: $18\pi a^2 - \pi a^2 = 17\pi a^2$	**B1**	Both areas, the disc's subtracted; may be implied by the moment equations.
	Semicircle $\left(0, \frac{4(6a)}{3\pi}\right) = \left(0, \frac{8a}{\pi}\right)$, disc $(2a, 2a)$	**B1**	Both centres of mass, soi.
	$17\pi a^2\bar{x} = 18\pi a^2(0) - \pi a^2(2a)$	**M1**	Forms a moment equation about the y-axis with their areas and the disc's x-coordinate; the semicircle's zero term need not appear. Condone sign slips.
	$\bar{x} = -\frac{2a}{17}$	**A1**	
	$17\pi a^2\bar{y} = 18\pi a^2\left(\frac{8a}{\pi}\right) - \pi a^2(2a)$	**M1**	Forms a moment equation about the x-axis with terms for both pieces, using their areas and distances. Condone sign slips.
	$\bar{y} = \frac{(144-2\pi)a}{17\pi}$	**A1**	
		[6]	
5(b)	$6a - \bar{x} = \frac{104a}{17}$ along RP $\bar{y} = \frac{(144-2\pi)a}{17\pi}$ perpendicular to RP	**B1ft**	Both distances from R, ft their $\bar{x}$ and $\bar{y}$ from (a); may be implied by the moments equation.
	$Mg\bar{y}\cos\theta = Mg(6a - \bar{x})\sin\theta$ $+ kMg(12a)\sin\theta$, $\tan\theta = \frac{1}{6}$	**M1 A1**	**M1** Takes moments about R for the lamina and the particle, resolving with the angle $\arctan\frac{1}{6}$, or places the system's centre of mass vertically below R. Condone sign slips. **A1** Correct equation, oe; may be in terms of $\bar{x}$ and $\bar{y}$.
	$k = \frac{216-29\pi}{51\pi}$	**A1**	
	$F = (M + kM)g$	**M1**	Equates the supporting force to the total weight, with their value of k.
	$F = \frac{216+22\pi}{51\pi}Mg$	**A1**	oe, or awrt $1.78Mg$.
		[6]	
Total for Question 5		**12**	

Question	Scheme	Marks	Notes
6(a)	$\frac{1}{2}\pi(4a)^2 = 8\pi a^2$, πa^2 Remaining area $7\pi a^2$	**B1**	Both areas, or their ratio, with the hole subtracted; soi in a moment equation.
	Semicircle's centre of mass $\left(0, \frac{16a}{3\pi}\right)$	**B1**	soi
	$7\pi a^2\bar{x} = 8\pi a^2(0) - \pi a^2(a) \Rightarrow \bar{x} = \ldots$	**M1**	Forms a moment equation for $\bar{x}$ with total, semicircle and hole terms, each area $\times$ distance; the semicircle's zero term need not appear. Condone sign slips.
	$\bar{x} = -\frac{a}{7}$	**A1**	Printed value reached from a correct equation, cso.
	$7\pi a^2\bar{y} = 8\pi a^2\left(\frac{16a}{3\pi}\right) - \pi a^2(2a)$	**M1**	Forms a moment equation for $\bar{y}$ with total, semicircle and hole terms, using their semicircle distance. Condone sign slips.
	$\bar{y} = \frac{(128-6\pi)a}{21\pi}$	**A1 AG**	Correct substituted equation leading without error to the printed $\bar{y}$; combined steps accepted.
		[6]	
6(b)	$8aT\cos 30° = 7g \times \frac{(128-6\pi)a}{21\pi}$	**M1 A1**	**M1** Takes moments about A for the tension (a component $\times 8a$, or T $\times$ perpendicular distance) and $7g$ $\times$ a coordinate of G. Condone sin/cos confusion. **A1** Correct equation, oe.
	$T \approx 16.4\,\text{N}$	**A1**	awrt 16 or 16.4
		[3]	
6(c)	$R_x + T\cos 30° = 0$ $R_y - T\sin 30° - 7g = 0$	**M1**	Resolves horizontally and vertically, each equation with all relevant forces. Condone sign slips and sin/cos confusion.
	$R_x = -T\cos 30°$ $[= -14.2]$ $R_y = 7g + T\sin 30°$ $[= 76.8]$	**A1**	Both correct, in terms of T or awrt 14.2 and 76.8 in size; either sign convention.
	$R = \sqrt{\text{“14.2”}^2 + \text{“76.8”}^2} = \ldots$ $\tan\theta = \frac{\text{“14.2”}}{\text{“76.8”}} \Rightarrow \theta = \ldots$	**M1**	Uses Pythagoras for the magnitude or a trigonometric ratio of their components for an angle. Condone the ratio inverted. Each A1 below needs its own method, seen or implied by its correct value.
	$R \approx 78.1\,\text{N}$	**A1**	awrt 78 or 78.1

Question 6 continued

Question	Scheme	Marks	Notes
	$\theta \approx 10.5°$, to the left of the upward vertical	**A1**	awrt $10.5°$ from the upward vertical, with its side stated as left or away from the arc, oe.
		[5]	
	Total for Question 6	**14**	

Question	Scheme	Marks	Notes
7(a)	$\frac{4\times3}{3\pi}$	**M1**	Substitutes $r=3$ into $\frac{4r}{3\pi}$.
	$=\frac{4}{\pi}\approx1.27\,\text{cm}$	**A1 AG**	Exact $\frac{4}{\pi}$ stated, and a decimal such as 1.273 seen before the printed 1.27.
		[2]	
7(b)(i)	Areas: 108 and $\frac{9\pi}{2}$	**B1**	
	Distances from AB: 9 and 6	**B1**	
	$\left(108+\frac{9\pi}{2}\right)\bar{x}=108\times9+\frac{9\pi}{2}\times6$	**M1**	Equates total area times $\bar{x}$ to the sum of area times distance from AB for both components. Condone slips in their areas or distances.
	$\bar{x}=\frac{216+6\pi}{24+\pi}$	**A1**	Any exact equivalent.
		[4]	
7(b)(ii)	Semicircle: $6+\frac{4}{\pi}$ from AD	**B1ft**	ft 6 plus their exact distance from (a).
	$\left(108+\frac{9\pi}{2}\right)\bar{y}=108\times3+\frac{9\pi}{2}\left(6+\frac{4}{\pi}\right)$	**M1**	Equates total area times $\bar{y}$ to the sum of area times distance from AD for both components. Condone slips in their areas or distances.
	$\bar{y}=\frac{76+6\pi}{24+\pi}$	**A1**	Any exact equivalent.
		[3]	
7(c)	Moment of tension about A: $10\left[18\sin(\theta+30°)+6\cos(\theta+30°)\right]$	**B1**	Any correct form, such as from horizontal and vertical components at C.
	Horizontal distance of centre of mass from A: $\bar{x}\cos\theta-\bar{y}\sin\theta$	**B1**	oe, with $\bar{x}$, $\bar{y}$ or their values from (b).
	$10\left[18\sin(\theta+30°)+6\cos(\theta+30°)\right]$ $=24(\bar{x}\cos\theta-\bar{y}\sin\theta)$	**M1 A1**	**M1** Equates moments about A of the tension and the weight, each a force times a perpendicular distance. Condone sign slips and sin and cos interchanged. **A1** Correct equation, with $\bar{x}$, $\bar{y}$ or their correct values.
	$\left(9\sqrt{3}-3+\frac{12\bar{y}}{5}\right)\sin\theta$ $=\left(\frac{12\bar{x}}{5}-9-3\sqrt{3}\right)\cos\theta$ $\Rightarrow\tan\theta=\ldots\ [\approx0.313]$	**dM1**	Expands both compound angles and collects terms to reach $\tan\theta=\ldots$, or solves their equation by another valid method. Condone sign slips.

Question 7 continued

Question	Scheme	Marks	Notes
	$\theta = 17.4°$	**A1**	awrt 17.4°.
		[6]	
	Total for Question 7	**15**	