Questions

1. The line ℓ_1 is the intersection of the planes

$$x + y + z = 1$$

and

$$2x - y + z = 4$$

The plane Π has equation

$$x + y + 2z = 3$$

The line ℓ_2 is the reflection of the line ℓ_1 in the plane Π.

Find a vector equation of the line ℓ_2. **[7]**

2. The plane Π has vector equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

The line ℓ_1 passes through the point $P(4, 1, 2)$ and is perpendicular to Π.
The line ℓ_1 meets Π at the point Q.

(a) Find the coordinates of Q. **[4]**

Given that the point $R(3, 1, 0)$ lies on Π,

(b) find a Cartesian equation of the plane containing P, Q and R. **[4]**

The line ℓ_2 passes through P and R.
The line ℓ_3 is the reflection of ℓ_2 in Π.

(c) Find a vector equation of ℓ_3. **[4]**

3. A plane passes through the points $C(1, -1, 2)$, $D(3, 0, 1)$ and $E(2, 2, 2)$.

The point A has coordinates $(4, -2, 7)$ and the point B is the reflection of A in the plane.

Find the coordinates of the point B. **[7]**

4. A line ℓ has equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

Another line m has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

A plane Π contains m and is parallel to ℓ.

(i) Find an equation of Π, giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = p$. [**4**]

(ii) Find the distance between ℓ and Π. [**4**]

(iii) Find an equation of the line which is the reflection of ℓ in Π, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [**4**]

5. The line ℓ_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

The plane Π_1 passes through the point $(2, 0, 0)$ and has normal vector

$$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

(a) Find the point of intersection of ℓ_1 and Π_1. **[2]**

The line ℓ_2 is the reflection of the line ℓ_1 in the plane Π_1.

(b) Show that a vector equation for the line ℓ_2 is

$$\mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

where μ is a scalar parameter. **[5]**

The plane Π_2 contains the line ℓ_1 and the line ℓ_2.

(c) Determine a vector equation for the line of intersection of Π_1 and Π_2. **[2]**

6. The triangle ABC lies in the plane Π. The vertices are

$$A(1,0,0),\ B(2,2,1) \text{ and } C(1,1,c)$$

where c is a constant.

(a) Find, in terms of c, $\overrightarrow{BA} \times \overrightarrow{BC}$. **[3]**

Given that $\overrightarrow{BA} \times \overrightarrow{BC} = d\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, where d is a constant,

(b) find the value of c and show that $d = -3$. **[2]**

(c) find an equation of Π in the form $\mathbf{r} \cdot \mathbf{n} = p$, where p is a constant. **[3]**

The point S has position vector $3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. The point S' is the image of S under reflection in Π.

(d) Find the position vector of S'. **[5]**

7. The line ℓ has equation

$$\mathbf{r} = (\mathbf{i} + \mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$$

where λ is a scalar parameter, and the plane Π has equation

$$\mathbf{r} \cdot (2\mathbf{i} - \mathbf{j} + \mathbf{k}) = 9$$

(a) Find the coordinates of the point of intersection of ℓ and Π. **[4]**

The perpendicular to Π from the point $A\,(0, 4, 1)$ meets Π at the point B.

(b) Verify that the coordinates of B are $(4, 2, 3)$. **[3]**

The point $A\,(0, 4, 1)$ is reflected in the plane Π to give the image point A'.

(c) Find the coordinates of the point A'. **[2]**

(d) Find an equation for the line obtained by reflecting the line ℓ in the plane Π, giving your answer in the form

$$\mathbf{r} \times \mathbf{a} = \mathbf{b}$$ **[4]**