Questions

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1. Three planes have equations

$$\begin{aligned} x - y + 2z &= 3 \\ 2x + y + z &= 4 \\ 3x + (m+1)z &= 7 \end{aligned}$$

where m is a constant.

(i) Investigate the arrangement of the planes:
when $m = 2$;
when $m \neq 2$. **[6]**

(ii) A student claims that the position vectors $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{k}$ all lie in a plane through the origin. Determine whether or not the student is correct. **[2]**

2. **(a)** Consider the linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 3 & -2 & 1 \\ 2 & 1 & -4 \\ 7 & 0 & -7 \end{pmatrix}, \qquad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 5 \\ 1 \\ 7 \end{pmatrix}$$

Show that A is singular, and deduce that the system does not have a unique solution. **[2]**

(b) Show that the system is consistent. Interpret the solution set geometrically. **[3]**

3. **(a)** Show that the system of equations

$$x + 2y - z = 3$$
$$2x - y + 4z = 1$$
$$3x + y + 3z = a$$

where a is a constant, does not have a unique solution. **[2]**

(b) Given that $a = 4$, show that the system of equations in part (a) is consistent. Interpret this situation geometrically. **[3]**

(c) Given instead that $a \neq 4$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. **[2]**

4. The equations

$$x - y + 2z = 4$$
$$(k+1)x + ky + (2-k)z = 4 - k$$
$$(5-2k)x + (2-2k)y + (k+2)z = 8 - k$$

represent three planes, where k is a real constant.
The planes do not intersect in exactly one point.

(a) Find the possible values of k. **[3]**

(b) For each value of k found in part (a), describe the configuration of the planes.
Fully justify your answer, stating in each case whether the corresponding system is consistent or inconsistent. **[5]**

5. Three planes have equations

$$x + ay = -1$$
$$2x + y + z = 3$$
$$3x + by + 2z = c$$

where a, b and c are constants.

(a) In the case where the planes **do not** intersect at a unique point,

(i) find b in terms of a **[4]**

(ii) find the value of c for which the planes form a sheaf. **[3]**

(b) In the case where $b = 1 - a$ and $c = 5$, find the coordinates of the point of intersection of the planes in terms of a. **[6]**

6.

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 0 \\ 3 & k & 1 \\ 2 & -1 & 1 \end{pmatrix}$$

where k is a constant.

(a) Find the values of k for which the matrix $\mathbf{M}$ has an inverse. **[2]**

(b) Find, in terms of p, the coordinates of the point where the following planes intersect:

$$\begin{aligned} x + y &= 1 \\ 3x + 2y + z &= p \\ 2x - y + z &= 4 \end{aligned}$$

[5]

(c) (i) Find the value of q for which the set of simultaneous equations

$$\begin{aligned} x + y &= 1 \\ 3x + z &= q \\ 2x - y + z &= 4 \end{aligned}$$

[2]

has at least one solution.

(ii) For this value of q, interpret the solution set geometrically. **[2]**

7. You are given that a is a parameter which can take only real values.
The matrix A is given by

$$A = \begin{pmatrix} 3 & 2 & a \\ 1 & 3 & 0 \\ 2 & -1 & 1 \end{pmatrix}$$

(a) Find an expression for the determinant of A in terms of a. **[2]**

You are given the following system of equations in x, y and z.

$$\begin{aligned} 3x + 2y + az &= 8 \\ x + 3y &= 5 \\ 2x - y + z &= 3 \end{aligned}$$

The system can be written in the form

$$A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ 3 \end{pmatrix}$$

(b) (i) In the case where A is not singular, solve the given system by using A^{-1}. **[5]**

(ii) In the case where A is singular, describe the configuration of the planes whose equations are the three equations of the system. **[3]**

The transformation represented by A is denoted by T.
A 3-D solid of volume $a^2 + 3$ is transformed by T to an image.

(c) (i) Determine the range of values of a for which the orientation of the image is the reverse of the orientation of the object. **[1]**

(ii) Determine the range of values of a for which the volume of the image is less than the volume of the object. **[2]**