Questions

1.

$$B = \begin{pmatrix} 1 & a \\ 1 & 3 \end{pmatrix}$$

where a is a real constant and $a \neq 3$.

(a) Find B^{-1} in terms of a. **[3]**

Given that

$$B^{-1} = 2\mathbf{I} - \frac{1}{2}B$$

where $\mathbf{I}$ is the 2×2 identity matrix,

(b) find the value of a. **[3]**

2. Given that

$$A = \begin{pmatrix} k+1 & 2 \\ 1-2k & k-4 \\ 2k-1 & 4-k \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

where k is a constant,

(a) determine the matrix BA. **[2]**

(b) hence find the values of k for which BA is singular. **[3]**

3. A community centre offers three evening courses: art, dance and pottery. At the start of the year there were 580 enrolments in total. The number enrolled in art was 20 more than the total number enrolled in dance and pottery combined.

By the end of the year:

- the number enrolled in art had decreased by 5%
- the number enrolled in dance had increased by 10%
- the number enrolled in pottery had increased by 15%
- overall enrolment had increased by 20

Form and solve a matrix equation to find the initial number enrolled in each course. **[7]**

4. The matrix A is given by

$$A = \begin{pmatrix} 2 & k & 1 \\ 1 & 0 & 2 \\ k & -1 & 1 \end{pmatrix}$$

where k is a real constant.

(a) Show that A is non-singular for all real values of k. **[3]**

(b) Given that

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \end{pmatrix}$$

find the value of k. **[2]**

5. A cinema chain sells three types of annual pass, A, B and C.

It is known that pass A makes a profit of £16 per pass sold, pass B makes a profit of £15 per pass sold and pass C makes a profit of £12 per pass sold.

Between 2023 and 2024

- sales of pass A increased by 25%
- sales of pass B decreased by 20%
- sales of pass C increased by 50%

In total 900 000 passes were sold in 2023.

In total 1 050 000 passes were sold in 2024.

The total profit from these passes in 2024 was £15.12 million.

Form and solve a matrix equation to find, to 2 significant figures, the number of each type of pass sold in 2023. **[8]**

6.

$$A = \begin{pmatrix} a & 1 & 0 \\ 2 & 3 & 1 \\ 0 & 1 & a \end{pmatrix}$$

where a is a real constant.

(a) Determine the values of a for which A is singular. **[2]**

(b) Hence, for all other values of a, find A^{-1} in terms of a. **[4]**

7. You are given that a is a real parameter.

The matrix A is given by

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 1 & -1 \\ a & 0 & 1 \end{pmatrix}$$

(a) Find an expression for $\det(A)$ in terms of a. **[2]**

You are given the following system of equations in x, y and z.

$$\begin{aligned} x - 2y + 3z &= -6 \\ 4x + y - z &= 7 \\ ax + z &= a - 1 \end{aligned}$$

The system can be written in the form

$$A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -6 \\ 7 \\ a - 1 \end{pmatrix}$$

(b) (i) In the case where A is not singular, solve the given system by using A^{-1}. **[5]**

(ii) In the case where A is singular, describe the configuration of the planes whose equations are the three equations of the system. **[3]**

The transformation represented by A is denoted by T.
A solid of volume 12 is transformed by T to an image in 3-D space.

(c) (i) Determine the range of values of a for which the orientation of the image is the reverse of the orientation of the object. **[1]**

(ii) Determine the range of values of a for which the volume of the image is less than the volume of the object. **[2]**

8.

$$A = \begin{pmatrix} k & 2 & 1 \\ -1 & 3 & 0 \\ 4 & -2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & -4 & -3 \\ 1 & k-4 & -1 \\ -10 & 2k+8 & 3k+2 \end{pmatrix}$$

where k is a constant.

(a) Determine the value of the constant c for which

$$AB = (3k - c)I$$ **[2]**

(b) Hence determine the value of k for which A^{-1} does not exist. **[2]**

Given that A^{-1} does exist,

(c) write down A^{-1} in terms of k. **[1]**

(d) Use the answer to part (c) to solve the simultaneous equations

$$\begin{aligned} kx + 2y + z &= 1 \\ -x + 3y &= 2 \\ 4x - 2y + z &= 3 \end{aligned}$$

giving the values of x, y and z in simplest form in terms of k. **[3]**

9. A company manufactures only three models of bicycle: Commuter, Electric and Folding.

Each bicycle manufactured is one of these three models.

In 2023, a total of 1000 bicycles were manufactured.

The number of Folding bicycles manufactured was 200 fewer than the combined number of Commuter and Electric bicycles.

In 2024 the number manufactured of

- Commuter increased by 10%
- Electric decreased by 5%
- Folding increased by 5%

In 2024, the total number of bicycles manufactured increased by 3.8%.

(a) (i) Define, for each model, a variable for the number of bicycles manufactured in 2023. **[2]**

(ii) Using your variables from part (a)(i), write down three equations that model this situation. **[2]**

(b) By forming and solving a matrix equation, determine how many bicycles of each model were manufactured in 2023. **[4]**

10. Three planes have equations

$$x + 2y - z = 4,$$
$$3x + ky + 2z = k + 11,$$
$$ky + 4z = k + 4$$

where k is a constant.

(a) Show that the planes have a unique point of intersection except for one value of k, which should be found. **[4]**

(b) Show that, whenever the planes have a unique point of intersection, this point is independent of k. Find the point. **[6]**