CORE PURE / MISCELLANEOUS PURE

Standard Summations

Questions by topic 15 questions · 120 marks

EXAM BOARDS Edexcel 9FM0 · AQA 7367 · OCR A H245 · OCR MEI H645 · CIE 9231

Questions

Question 1	*4 marks*	2
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1. Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that

$$\sum_{r=5}^{12}(2r-1)(r+3) = 1556 \qquad \textbf{[4]}$$

2. Using standard summation results, find the exact value of

$$\sum_{r=21}^{60} (3r-1)(r+2) \qquad \textbf{[4]}$$

3. Using standard summation results, find the exact value of

$$\sum_{r=11}^{40} r(r-3)(r+3) \qquad \textbf{[4]}$$

4. The positive integer k is such that

$$\sum_{r=1}^{k} \left(6r^2 - k^2\right) = 205$$

Find the value of k, showing clearly that there is only one possible value. **[4]**

5. **(a)** Prove that, for all positive integers n,

$$\sum_{r=1}^{n}(2r-1)^3 = n^2\left(2n^2-1\right)$$ **[5]**

(b) Jo evaluated

$$\frac{1}{n^2}\sum_{r=1}^{n}(2r-1)^3$$

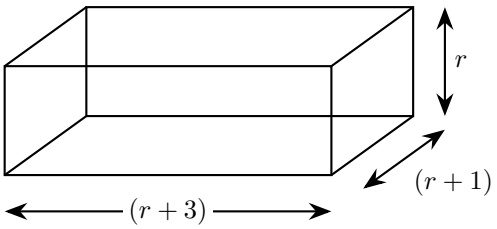
for $n = 2$, $n = 3$ and $n = 4$, and obtained a prime number each time. Jo then made the statement:

"For all integers $n \geq 2$,

$$\frac{1}{n^2}\sum_{r=1}^{n}(2r-1)^3$$

is a prime number."

Disprove Jo's statement. **[2]**



6. A block in the shape of a cuboid has length $(r+3)$ cm, width $(r+1)$ cm and height r cm, as shown in the diagram above.

A set consists of n such blocks. The first block has a height of 1 cm, the second block has a height of 2 cm, the third block has a height of 3 cm, and so on, so that the rth block has height r cm.

(a) Use the standard results for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that the **total** volume, V cm^3, of all n blocks in the set is given by

$$V = \frac{n}{12}(n+1)(n+2)(3n+13), \qquad n \geq 1$$ **[5]**

(b) Find the smallest number of blocks in the set for which the total volume of the set exceeds 5000 cm^3. **[2]**

7. **(a)** Use the standard results for $\sum r$, $\sum r^2$ and $\sum r^3$ to show that for all positive integers n

$$\sum_{r=1}^{n} r(2r-1)^2 = \frac{1}{6}n(n+1)(an^2+bn+c)$$

where a, b and c are integers to be determined. **[5]**

(b) Hence determine the positive integer k for which

$$\sum_{r=1}^{k} r(2r-1)^2 = 193\sum_{r=1}^{k} r$$ **[4]**

8. **(a)** Show that

$$\sum_{r=1}^{n} r^2(r+3)$$

can be expressed in the form

$$kn(n+1)(an^2+bn+c)$$

where k is a rational number and a, b and c are integers. **[4]**

(b) Show that there is exactly one positive integer n for which

$$2\sum_{r=1}^{n} r^2(r+3) = 19\sum_{r=1}^{n} r(r+1)$$

and state this value of n. **[6]**

9. **(a)** Find an expression for

$$\sum_{r=1}^{2m}(r-1)(r+3)$$

in the form $\frac{1}{3}m(am^2+bm+c)$, where a, b, c are integers whose values are to be determined. **[4]**

(b) Hence show that

$$\sum_{r=2m+1}^{4m}(r-1)(r+3)=\frac{1}{3}m\left(56m^2+54m-11\right)$$

and use this result to find the value of

$$\sum_{r=21}^{40}(r-1)(r+3)$$ **[5]**

10. **(i)** Use standard series to show that

$$\sum_{r=1}^{n} r(r+1)(4r-k) = \frac{1}{3}n(n+1)(n+2)(3n+1-k)$$

where k is a constant. **[5]**

(ii) Using your result from part (i) with $k = 1$, find

$$\sum_{r=n+1}^{2n} r(r+1)(4r-1)$$

giving your answer in fully factorised form. **[3]**

11. **(a)** Use the standard results for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n}(2r-1)^3 = n^2\left(2n^2-1\right)$$

for all positive integers n. **[5]**

(b) Hence find any positive integer values of n for which

$$\sum_{r=n+1}^{2n}(2r-1)^3 = 29n^4 - 34n^2 + \sum_{r=1}^{\infty} 240\left(\frac{3}{5}\right)^r$$ **[5]**

12. **(a)** Use the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r+1)(2r+1) = \frac{1}{2}n(n+a)^2(n+b)$$

where a and b are integers to be found. **[5]**

(b) Given that

$$\sum_{r=11}^{n} r(r+1)(2r+1) = 17\,940$$

use your answer to part (a) to find the value of the positive integer n, showing clear algebraic working. **[4]**

13. In this question you may assume the results for

$$\sum_{r=1}^{n} r^3, \quad \sum_{r=1}^{n} r^2 \quad \text{and} \quad \sum_{r=1}^{n} r$$

(a) Show that

$$1^3 - 2^3 + 3^3 - 4^3 + \cdots + (2n-1)^3 - (2n)^3 = -n^2(4n+3) \qquad \textbf{[5]}$$

Ten consecutive positive integers are written down, the smallest of which is odd. Starting with the cube of the smallest integer, the cubes of these ten integers are alternately added and subtracted, so that the cube of the largest integer is subtracted. The result is -3725.

(b) Use the answer to part (a) to determine the smallest of these ten integers. **[4]**

14. **(a)** Use the standard summation formulae to show that, for $n \in \mathbb{N}$,

$$\sum_{r=1}^{n} \left(6r^2 + 2r - 8\right) = an(n-b)(n+c)$$

where a, b and c are positive integers to be determined. **[4]**

(b) Explain why, for $k \in \mathbb{N}$,

$$\sum_{r=1}^{4k+1} r \sin{(90r)}^\circ = 2k + 1$$ **[3]**

Using the results from part (a) and part (b) and showing all your working,

(c) find the value of the integer n, where $n \geq 3$, that satisfies

$$\sum_{r=3}^{n} \left(6r^2 + 2r - 8\right) = 3\left[\sum_{r=3}^{4n+1} r \sin{(90r)}^\circ\right]^2$$

and show that there are no other such values of n. **[6]**

15. **(a)** Explain why, for $n \in \mathbb{N}$,

$$\sum_{r=1}^{2n}(-1)^r f(r) = \sum_{r=1}^{n}\left(f(2r) - f(2r-1)\right)$$

for any function $f(r)$. **[2]**

(b) Use the result of part (a) and the standard summation formulae to show that, for $n \in \mathbb{N}$,

$$\sum_{r=1}^{2n}(-1)^r r^2(r+1) = n(n+1)(4n+1)$$ **[4]**

(c) Hence show that, for $n \in \mathbb{N}$,

$$\sum_{r=1}^{2n} r(r+1)\left((-1)^r r + 3\right) = n(n+1)(12n+5)$$ **[3]**

(d) Hence evaluate

$$\sum_{r=11}^{41} r(r+1)\left((-1)^r r + 3\right)$$ **[4]**