Questions

1. A curve has equation

$$y = 2 - \frac{3}{x+1}$$

(a) State the equations of the horizontal and vertical asymptotes of this curve. **[2]**

(b) The straight line $y = 3x + c$ meets the curve. Show that the x-coordinates of the points of intersection satisfy

$$3x^2 + (c+1)x + (c+1) = 0$$

[3]

(c) It is given that the line $y = 3x + c$ is a tangent to the curve.

(i) Without using calculus, find the two possible values of c. **[3]**

(ii) Hence find the coordinates of each point of contact. **[4]**

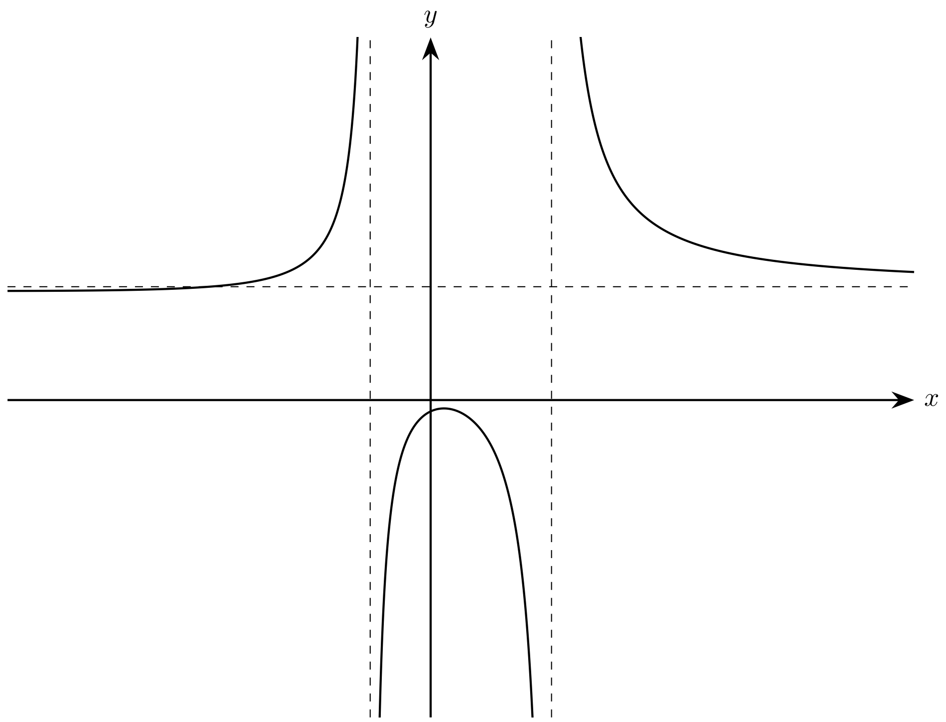
2. The curve C has equation $y = \dfrac{x+1}{x^2 - 4x + 3}$.

(a) Find the equations of the asymptotes of C. **[2]**

(b) Find the coordinates of any stationary points on C. **[4]**

(c) Sketch C, stating the coordinates of the intersections with the axes. **[3]**

(d) Sketch the curve with equation $y = \left|\dfrac{x+1}{x^2 - 4x + 3}\right|$ and find in exact form the set of values of x for which $2|x+1| > |x^2 - 4x + 3|$. **[6]**



3. The diagram shows the curve C with equation

$$y = \frac{5x^2 + ax - b}{x^2 + bx + a}$$

where a and b are integers.

The vertical asymptotes of C are $x = -1$ and $x = 2$.

(a) State the equation of the asymptote of C that is parallel to the x-axis. **[1]**

(b) Use the vertical asymptotes to find the value of a and the value of b. **[2]**

(c) Hence, or otherwise, find the coordinates of the point where C meets the y-axis. **[1]**

(d) Without using calculus, show that the line $y = 4$ does not intersect C. **[5]**

4. The function f is defined by

$$f(x) = \frac{-2x^2 + 11x - 20}{2x - 3} \qquad (x \in \mathbb{R},\ x \neq \tfrac{3}{2})$$

(a) Find the range of f **[5]**

(b) Find the coordinates of the two stationary points of the graph of $y = f(x)$. **[2]**

(c) Show that the graph of $y = f(x)$ has an oblique asymptote and find its equation. **[2]**

(d) Sketch the graph of $y = f(x)$, showing clearly the asymptotes and stationary points. **[4]**

5. The curve C has equation $y = \dfrac{x^2 + ax - 6}{x + 1}$, where $a > 0$.

(a) Find the equations of the asymptotes of C. **[3]**

(b) Show that C has no stationary points. **[4]**

(c) Sketch C, stating the coordinates of the point of intersection with the y-axis and labelling the asymptotes. **[3]**

(d) (i) Sketch the curve with equation $y = \left|\dfrac{x^2 + ax - 6}{x + 1}\right|$. **[2]**

(ii) On your sketch in part (i), draw the line $y = a + 1$. **[1]**

(iii) It is given that $\left|\dfrac{x^2 + ax - 6}{x + 1}\right| < a + 1$ for $-11 < x < -3$ and $0 < x < 4$.

Find the value of a. **[2]**

6. The function f is defined by

$$f(x) = \frac{x^2 + 2x - 3}{x^2 + px + 4} \qquad x \in \mathbb{R}$$

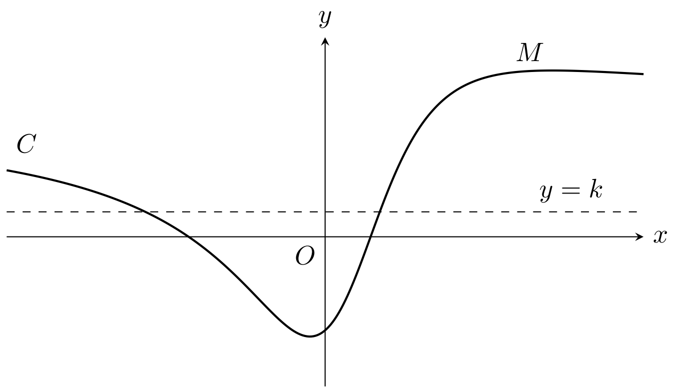
where p is a constant.

The graph of $y = f(x)$ has only one asymptote.

(a) Write down the equation of the asymptote. **[1]**

(b) Find the set of possible values of p. **[4]**

(c) Find the coordinates of the points at which the graph of $y = f(x)$ intersects the coordinate axes. **[3]**



(d) A curve C has equation

$$y = \frac{x^2 + 2x - 3}{x^2 - x + 4}$$

The curve C has a local maximum at the point M as shown in the diagram.
The line $y = k$ intersects curve C.

(i) Show that

$$15k^2 - 8k - 16 \leq 0$$ **[5]**

(ii) Hence, find the y-coordinate of point M. **[2]**

7. A curve has equation

$$y = \frac{x^2 - 9}{x^2 - 4}$$

(a) Write down the equations of the asymptotes to the curve. **[3]**

(b) Sketch the curve, indicating the coordinates of the points where the curve intersects the coordinate axes. **[5]**

(c) Hence, or otherwise, solve the inequality

$$0 < \frac{x^2 - 9}{x^2 - 4} < 1$$

[2]

8. The curve C has equation

$$y = \frac{x^2+3}{x+1}$$

(a) Find the equations of the asymptotes of C. **[3]**

(b) Show that there is no point on C for which $-6 < y < 2$. **[4]**

(c) Sketch C. **[2]**

(d) (i) Sketch the graphs of

$$y = \left|\frac{x^2+3}{x+1}\right| \text{ and } y = |x-1|$$

on the same axes, stating the coordinates of any intersections with the axes. **[4]**

(ii) Use your sketch to find the set of values of c for which

$$\left|\frac{x^2+3}{x+1}\right| \leq |x-1| + c$$

has no solution. **[1]**

9. Let a be a positive constant.

(a) The curve C_1 has equation

$$y = \frac{x^2 - a^2}{x^2 - 4a^2}$$

Sketch C_1. **[2]**

The curve C_2 has equation

$$y = \left(\frac{x^2 - a^2}{x^2 - 4a^2}\right)^2$$

The curve C_3 has equation

$$y = \left|\frac{x^2 - a^2}{x^2 - 4a^2}\right|$$

(b) (i) Find the coordinates of all stationary points of C_2. **[3]**

(ii) Find also the coordinates of all points of intersection of C_2 and C_3. **[3]**

(c) Sketch C_2 and C_3 on a single diagram, clearly identifying each curve. Hence find the set of values of x for which

$$\left(\frac{x^2 - a^2}{x^2 - 4a^2}\right)^2 \leq \left|\frac{x^2 - a^2}{x^2 - 4a^2}\right|$$ **[5]**

10. A curve C_1 has equation

$$y = -2 + \frac{12}{x+3}$$

(a) Write down the equations of the asymptotes of curve C_1. **[2]**

(b) Sketch the graph of curve C_1. Indicate the values of the intercepts of the curve with the axes. **[3]**

(c) Hence, or otherwise, solve the inequality

$$-2 + \frac{12}{x+3} > 0$$

[2]

(d) Curve C_2 is a reflection of curve C_1 in the line $y = -x$. Find an equation for curve C_2 in the form $y = f(x)$. **[3]**

11. The curve C has equation

$$y = \frac{x^2 - 6x + 5}{x + 2}$$

(a) Find the equations of the asymptotes of C. **[3]**

(b) Find the exact coordinates of the stationary points on C. **[4]**

(c) Sketch C, stating the coordinates of any intersections with the axes. **[3]**

(d) Sketch the curve with equation

$$y = \left| \frac{x^2 - 6x + 5}{x + 2} \right|$$

and find in exact form the set of values of x for which

$$\left| \frac{x^2 - 6x + 5}{x + 2} \right| < 6$$

[5]

12. A curve has equation $y = \dfrac{x-1}{x^2-x-6}$.

(a) (i) Write down the equations of the three asymptotes of the curve. **[3]**

(ii) Sketch the curve, showing the coordinates of any points of intersection with the coordinate axes. **[4]**

(b) Hence, or otherwise, solve the inequality

$$\frac{x-1}{x^2-x-6} > \frac{1}{2}$$

[3]

13. The curve C has equation

$$y = \frac{(x-1)(x+3)}{x^2-2x+3}$$

(a) Show that C has no vertical asymptotes and state the equation of the horizontal asymptote of C. **[3]**

(b) Find the coordinates of the stationary points on C. **[4]**

(c) Sketch C, stating the coordinates of the intersections with the axes. **[3]**

(d) Sketch the curve with equation

$$y = \left|\frac{(x-1)(x+3)}{x^2-2x+3}\right|$$

and state the set of values of k for which

$$\left|\frac{(x-1)(x+3)}{x^2-2x+3}\right| = k$$

has 4 distinct real solutions. **[2]**

14. The curve C has equation

$$y = \frac{-x^2 + 4x + 5}{x + 2}$$

(a) Find the equations of the asymptotes of C. **[3]**

(b) Find the coordinates of the stationary points of C. **[4]**

(c) Sketch C, stating the coordinates of the intersections with the axes. **[3]**

(d) Sketch the curve with equation

$$y = \left| \frac{-x^2 + 4x + 5}{x + 2} \right|$$ **[2]**

(e) Using your sketch, or otherwise, find the set of values of x for which

$$\left| \frac{-x^2 + 4x + 5}{x + 2} \right| < 2$$ **[4]**