CORE PURE / MISCELLANEOUS PURE

Algebraic and Modulus Inequalities

Questions by topic 15 questions · 107 marks

EXAM BOARDS AQA 7367 · CIE 9231 · Edexcel FP1 9FM0/3A

Questions

Question 1	*5 marks*	2
Question 2	*6 marks*	3
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Question 4	*8 marks*	5
Question 5	*8 marks*	6
Question 6	*5 marks*	7
Question 7	*5 marks*	8
Question 8	*11 marks*	9
Question 9	*8 marks*	10
Question 10	*8 marks*	11
Question 11	*7 marks*	12
Question 12	*6 marks*	13
Question 13	*7 marks*	14
Question 14	*8 marks*	15
Question 15	*9 marks*	16

1. The function f is defined by

$$f(x) = |2\cos x - 1| \qquad (0 \leq x \leq 2\pi)$$

Find the set of values of x for which

$$f(x) < 1$$

Give your answer in set notation. **[5]**

2. **(a)** On the same axes, sketch the graphs of $y = |3x - 2|$ and $y = |x + 4|$, stating the coordinates of the points where each graph meets the coordinate axes. **[3]**

(b) Hence, or otherwise, solve the inequality

$$|3x - 2| > |x + 4|$$ **[3]**

3. **(a)** Use algebra to determine the values of x for which

$$\frac{4x+1}{2x+3} \leq -3 \qquad \textbf{[4]}$$

(b) Hence, by using a suitable substitution, find the values of y for which

$$\frac{4+y}{2+3y} \leq -3 \qquad \textbf{[2]}$$

4. The function f is defined by

$$f(x) = 9x^3 + 24x^2 - 44x + 16 \qquad (x \in \mathbb{R})$$

(a) (i) Fully factorise $f(x)$. **[2]**

(ii) Hence solve the inequality $f(x) > 0$. **[2]**

(b) The graph of $y = f(x)$ is reflected in the y-axis. The new graph is then translated by the vector

$$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

to give the graph of $y = g(x)$.
Write $g(x)$ as a product of linear factors, and hence solve the inequality $g(x) \leq 0$. **[4]**

5. The function f is defined by

$$f(x) = \frac{ax + b}{x - 2} \qquad x \in \mathbb{R},\ x \neq 2$$

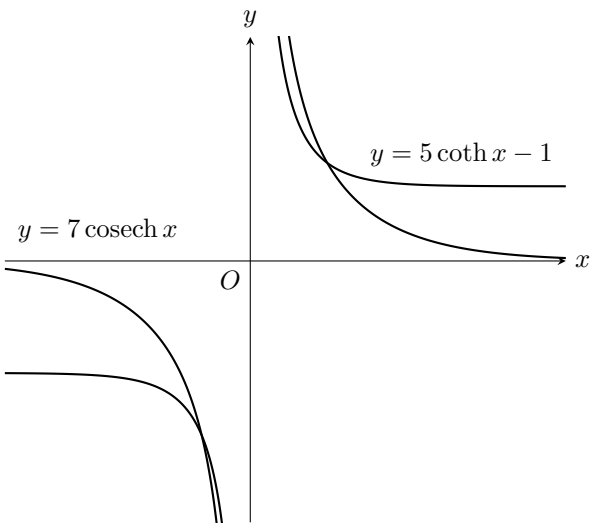
where a and b are integers.
The graph of $y = f(x)$ has a horizontal asymptote $y = 4$ and passes through the point $(3, 7)$.

(a) Find the value of a and the value of b. **[2]**

(b) Solve the inequality

$$f(x) \geq 2 - x$$

Fully justify your answer. **[6]**



6. The diagram above shows part of the graph of $y = 7\,\text{cosech}\,x$ and part of the graph of $y = 5\coth x - 1$. Solve the inequality

$$7\,\text{cosech}\,x < 5\coth x - 1$$

giving your answer in logarithmic form. **[5]**

7. The integers a and b are such that the inequality

$$(x^2 + ax - 12)(x^2 - 2x + b) < 0$$

has the solution set

$$\{x \in \mathbb{R} \mid -2 < x < -1\} \cup \{x \in \mathbb{R} \mid 3 < x < c\}$$

Find the values of a, b and c. **[5]**

8. An engineering student is testing two small robots, A and B, which move along a straight track. The distances, in centimetres, of robots A and B from a fixed marker O on the track are modelled by

$$d_A = |3t - 25|$$
$$d_B = \left|2t^2 - 15t + 3\right|$$

respectively, where t is the time in seconds after the robots are switched on. The model is used for $0 \leq t \leq 10$.

(a) Use algebra to find the set of values of t, with $0 \leq t \leq 10$, for which robot B is further from O than robot A is. **[8]**

In a trial, robot B was recorded as being further from O than robot A for a total of approximately 6 seconds during the first 10 seconds.

(b) Find, in exact form, the total length of time during the first 10 seconds for which the model predicts that robot B is further from O than robot A, and use your answer to assess the validity of the model. **[3]**

9. The function f is defined by

$$f(x) = \frac{x^2 + ax + 2}{x + 3}, \qquad x \neq -3$$

where a is a constant and $a > \frac{11}{3}$.

(a) Sketch the graph of $y = |f(x)|$. **[2]**

(b) On your sketch in part (a), draw the line $y = a$. **[1]**

(c) It is given that the line $y = a$ meets the graph of $y = |f(x)|$ at the point where $x = 4$.
Find the value of a and hence find the set of values of x for which $|f(x)| < a$. **[5]**

10. Given that k is a positive constant and that

$$\frac{kx}{x+3} \leq \frac{x-1}{x-2}$$

(a) show that

$$(x-2)(x+3)(px^2+qx+r) \leq 0$$

where p, q and r are expressions in terms of k to be determined. **[3]**

(b) Given also that $px^2+qx+r=0$ when $x=3$, find the value of k.
Hence, or otherwise, find the values of x for which the given inequality holds for this value of k. **[5]**

11. Let a be a positive constant.

(a) Sketch the curve with equation $y = \frac{2x-a}{x+a}$, stating the equations of its asymptotes and the coordinates of the points where it meets the coordinate axes. **[3]**

(b) Sketch the curve with equation $y = \left|\frac{2x-a}{x+a}\right|$ and find, in terms of a, the set of values of x for which $\left|\frac{2x-a}{x+a}\right| > 3$. **[4]**

12. The curve C has equation

$$y = \frac{x-3}{2-x-x^2}$$

Sketch the curve with equation

$$y = \left|\frac{x-3}{2-x-x^2}\right|$$

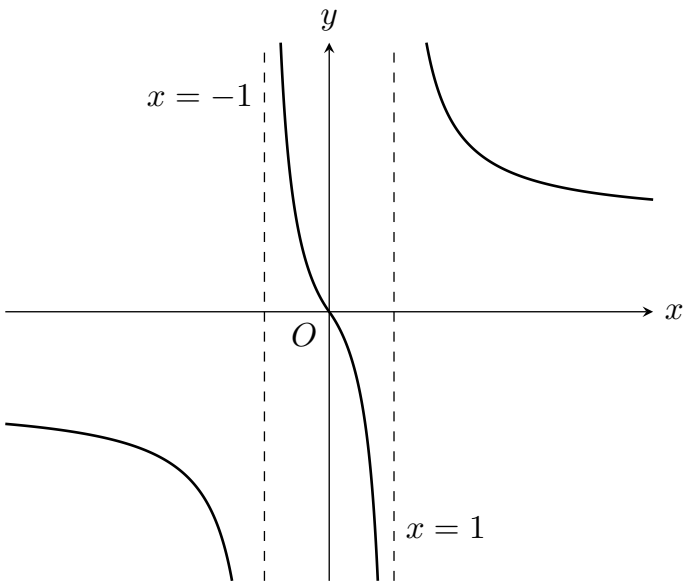
and find, in exact form, the set of values of x for which $|x-3| > 2\left|2-x-x^2\right|$. **[6]**

13. The curve C has equation $y = f(x)$, where

$$f(x) = \frac{x^2 + 3}{x^2 + 2x - 3}$$

(a) Sketch the curve with equation $y = \frac{1}{f(x)}$, stating the coordinates of any points where it meets the coordinate axes and the equation of its asymptote. **[3]**

(b) Find the set of values of x for which $\frac{1}{f(x)} \geq f(x)$. **[4]**

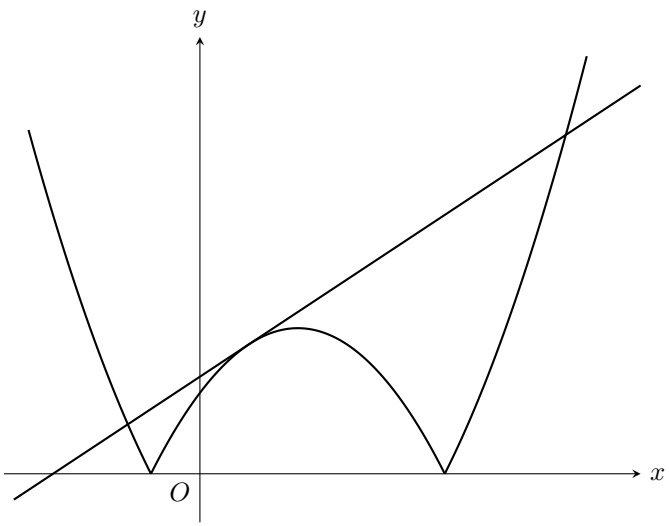


14. The diagram above shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \frac{2x}{|x| - 1}$$

Use algebra to determine the values of x for which

$$\frac{2x}{|x| - 1} \geq x + 3$$ **[8]**



15. The diagram above shows a sketch of the curve with equation $y = |x^2 - 4x - 5|$ and a sketch of the straight line with equation $y = mx + c$, where m and c are positive constants.
The equation

$$|x^2 - 4x - 5| = mx + c$$

has exactly 3 roots, as shown in the diagram above.

(a) Show that

$$m^2 - 8m - 4c + 36 = 0$$ **[2]**

Given that the line passes through the point $(-2, 2)$,

(b) determine the value of m and the value of c, justifying your answer. **[4]**

(c) Hence solve

$$|x^2 - 4x - 5| > mx + c$$ **[3]**