CORE PURE / MISCELLANEOUS PURE

Standard Summations

Mark scheme 15 questions · 120 marks

EXAM BOARDS Edexcel 9FM0 · AQA 7367 · OCR A H245 · OCR MEI H645 · CIE 9231

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	Total	**120**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1	$(2r-1)(r+3) = 2r^2+5r-3$ $\sum_{r=1}^{n} = \frac{1}{3}n(n+1)(2n+1) + \frac{5}{2}n(n+1) - 3n$	**M1 A1**	**M1** Expands to a quadratic in r and substitutes the standard results for $\sum r^2$ and $\sum r$, with general or numerical n. Condone slips in the expansion. **A1** Correct in any form, need not be simplified, including $-3n$. Also for correct substituted expressions at both $n = 12$ and $n = 4$.
	$\sum_{r=5}^{12} = \sum_{r=1}^{12} - \sum_{r=1}^{4}$ $=$ "1654" $-$ "98"	**M1**	Subtracts their sum to 4 from their sum to 12, both from the standard results. Not for subtracting the sum to 5.
	$= 1556$	**A1 AG**	Correct values for both sums from the formulae, leading without error to 1556.
	Total for Question 1	**4**	

Question	Scheme	Marks	Notes
2	$\sum(3r^2+5r-2)$ $= \frac{3}{6}n(n+1)(2n+1)+\frac{5}{2}n(n+1)-2n$	**M1**	Expands and replaces the r^2 and r terms by standard results, with general n or the limits directly. Condone a slip in the expansion or the constant term.
	$= n(n^2+4n+1)$	**A1**	oe, need not be factorised; or implied by correct sums to 60 and to 20.
	"$60(60^2+240+1)$" − "$20(20^2+80+1)$"	**M1**	Subtracts their sum to 20 from their sum to 60, from the same expression. Not for subtracting the sum to 21.
	$= 220\,840$	**A1**	
	Total for Question 2	**4**	

Question	Scheme	Marks	Notes
3	$r(r-3)(r+3) = r^3 - 9r$ $\sum_{r=1}^{n}(r^3 - 9r) = \frac{1}{4}n^2(n+1)^2 - \frac{9}{2}n(n+1)$	**M1 A1**	**M1** Expands to a polynomial in r and replaces each term by its standard result. Condone a slip in the expansion. **A1** Correct, oe, need not be simplified; implied by correct values for $n = 40$ and $n = 10$.
	Their sum to 40 minus their sum to 10: "665 020" − "2530"	**M1**	Substitutes $n = 40$ and $n = 10$ into their expression for a sum from $r = 1$ to n and subtracts. Not for $n = 11$.
	662 490	**A1**	
	Total for Question 3	**4**	

Question	Scheme	Marks	Notes
4	$6 \times \frac{1}{6}k(k+1)(2k+1) - k \times k^2$	**M1**	Uses $\frac{1}{6}k(k+1)(2k+1)$ for $\sum r^2$ and sums k^2 as k^3. Not for $\sum k^2$ taken as k^2 or as $\sum r^2$.
	$k^3 + 3k^2 + k - 205 = 0$	**A1**	oe, such as $k^3 + 3k^2 + k = 205$.
	$k = 5$ is a root $\Rightarrow (k-5)(k^2+8k+41) = 0$ $8^2 - 4 \times 1 \times 41 = \ldots$	**dM1**	Shows $k = 5$ is a root, then tests the quadratic factor by discriminant or completing the square, or argues the cubic increasing or the factor positive for $k > 0$. Condone slips.
	$-100 < 0$ so no real roots $\Rightarrow k = 5$ only	**A1**	cso. Needs the correct negative discriminant oe, or a correct increasing or positivity argument, and a conclusion that $k = 5$ is the only value. $k = 5$ by trial alone earns neither mark.
	Total for Question 4	**4**	

Question	Scheme	Marks	Notes
5(a)	$(2r-1)^3 = 8r^3 - 12r^2 + 6r - 1$ $\sum(2r-1)^3 = 8\sum r^3 - 12\sum r^2 + 6\sum r - \sum 1$	**M1**	Expands to four terms and writes the sum as multiples of $\sum r^3$, $\sum r^2$, $\sum r$ and $\sum 1$. Condone one coefficient slip.
	$= 8 \times \frac{1}{4}n^2(n+1)^2 - 12 \times \frac{1}{6}n(n+1)(2n+1)$ $+ 6 \times \frac{1}{2}n(n+1) - n$	**M1 A1**	**M1** Substitutes standard results for $\sum r^3$, $\sum r^2$ and $\sum r$ into their split sum. Condone $\sum 1$ taken as 1, or one slip in a formula. **A1** Correct expression; need not be simplified.
	$= n\big[2n^3 + 4n^2 + 2n - 4n^2 - 6n - 2$ $+ 3n + 3 - 1\big]$ $= \ldots$	**dM1**	Expands their expression and collects like terms, with or without a common factor taken out. Condone slips.
	$= n(2n^3 - n) = n^2(2n^2 - 1)$	**A1 AG**	Printed result reached without error from a correct substituted expression; combined steps accepted. Nothing for working backwards from the result.
		[5]	
5(b)	$\frac{1}{n^2}\sum_{r=1}^{n}(2r-1)^3 = 2n^2 - 1$ $n = 5 \Rightarrow 2 \times 5^2 - 1 = \ldots$	**M1**	Evaluates the expression for an integer $n \geq 5$, via $2n^2 - 1$ or summing directly.
	$49 = 7 \times 7$, not prime, so the statement is false	**A1**	Any counterexample, with a factor shown and a conclusion that the statement is false.
		[2]	
	Total for Question 5	**7**	

Question	Scheme	Marks	Notes
6(a)	$r(r+1)(r+3) = r^3 + 4r^2 + 3r$	**B1**	
	$V = \frac{1}{4}n^2(n+1)^2 + 4 \cdot \frac{1}{6}n(n+1)(2n+1)$ $+ 3 \cdot \frac{1}{2}n(n+1)$	**M1 A1**	**M1** Replaces each sum in their expansion by a standard result, at least two quoted correctly. Condone coefficient slips. **A1** Correct expression, unsimplified.
	$V = \frac{n(n+1)}{12}\left[3n(n+1) + 8(2n+1) + 18\right]$ $= \frac{n(n+1)}{12}\left(3n^2 + 19n + 26\right)$	**dM1**	Takes out a factor $n(n+1)$, or a multiple, and collects the rest into a quadratic; or expands fully to a quartic. Condone arithmetic slips.
	$V = \frac{n}{12}(n+1)(n+2)(3n+13)$	**A1 AG**	From a correct expression for V, reaches the printed result with no errors. Expanding both forms to the same quartic, with a conclusion, is accepted.
		[5]	
6(b)	$n = 10 : V = 4730 < 5000$ $n = 11 : V = 6578 > 5000$	**M1**	Evaluates V at consecutive n whose volumes lie either side of 5000 ($n = 10, 11$), or solves $V = 5000$ numerically.
	11 blocks	**A1**	
		[2]	
	Total for Question 6	**7**	

Question	Scheme	Marks	Notes
7(a)	$r(2r-1)^2 = 4r^3 - 4r^2 + r$	**B1**	
	$4 \times \frac{1}{4}n^2(n+1)^2 - 4 \times \frac{1}{6}n(n+1)(2n+1)$ $+ \frac{1}{2}n(n+1)$	**M1 A1**	**M1** Replaces each sum in their three-term expansion by its standard result. Condone one slip in a quoted result or coefficient. **A1** Any correct unsimplified equivalent.
	$\frac{1}{6}n(n+1)\left[6n(n+1) - 4(2n+1) + 3\right]$	**dM1**	Takes out $\frac{1}{6}n(n+1)$ or $n(n+1)$ and expands their bracket to a quadratic; expanding fully then factorising also accepted.
	$\frac{1}{6}n(n+1)\left(6n^2 - 2n - 1\right)$	**A1**	cso. The expression in the required form, or $a = 6$, $b = -2$, $c = -1$ stated.
		[5]	
7(b)	$\frac{1}{6}k(k+1)\left(6k^2 - 2k - 1\right) = 193 \times \frac{1}{2}k(k+1)$	**M1 A1**	**M1** Equates their result from (a) to 193 times the standard result for $\sum r$. Condone n for k. **A1** Correct equation in k, oe.
	$6k^2 - 2k - 1 = 579$ $\Rightarrow 3k^2 - k - 290 = 0 \Rightarrow k = \ldots$	**dM1**	Cancels $k(k+1)$, or a factor of it, to a three-term quadratic and solves it by any means, including a calculator.
	$k = 10$	**A1**	$k = 10$ only; A0 if $-\frac{29}{3}$ is kept as an answer.
		[4]	
	Total for Question 7	**9**	

Question	Scheme	Marks	Notes
8(a)	$\sum r^3 + 3\sum r^2$ $= \frac{1}{4}n^2(n+1)^2 + 3 \times \frac{1}{6}n(n+1)(2n+1)$	**M1 A1**	**M1** Writes the sum as $\sum r^3 + 3\sum r^2$ and substitutes standard results for both. Condone a slip in one result. **A1** Any correct unsimplified form.
	$= \frac{1}{4}n(n+1)\left[n(n+1) + 2(2n+1)\right]$	**dM1**	Simplifies their expression to a multiple of $n(n+1)$ times a quadratic, by taking out the common factor or otherwise. Condone arithmetic slips in the quadratic.
	$= \frac{1}{4}n(n+1)(n^2+5n+2)$	**A1**	cso. Any equivalent with rational k and integer a, b, c, e.g. $\frac{1}{8}n(n+1)(2n^2+10n+4)$.
		[4]	
8(b)	$\sum r(r+1) = \frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1)$	**M1**	Writes $\sum r(r+1)$ as $\sum r^2 + \sum r$ and substitutes both standard results. Condone a slip in one result.
	$= \frac{1}{3}n(n+1)(n+2)$	**A1**	Correct expression oe, e.g. $\frac{1}{3}n(n+1)(n+2)$; implied by a correct quadratic.
	$\frac{1}{2}n(n+1)(n^2+5n+2) = \frac{19}{3}n(n+1)(n+2)$ $\Rightarrow \frac{1}{2}(n^2+5n+2) = \frac{19}{3}(n+2)$	**M1**	Forms the equation from part (a) and their $\sum r(r+1)$, and divides by $n(n+1)$ or otherwise reduces it to a quadratic. Condone slips in constants.
	$3n^2 - 23n - 70 = 0$	**A1**	Any equivalent quadratic $= 0$.
	$(3n+7)(n-10) = 0 \Rightarrow n = \ldots$	**dM1**	Solves their three-term quadratic by factorising (product of constants equal to the constant term), the formula or completing the square, to $n = \ldots$; a calculator is allowed if both roots are stated.
	$n = 10$ $\left(n \neq -\frac{7}{3}\right)$	**A1**	cso. $n = 10$ only, with $-\frac{7}{3}$ rejected as not a positive integer. A0 without the rejection.
		[6]	
Total for Question 8		**10**	

Question	Scheme	Marks	Notes
9(a)	$\sum(r^2+2r-3)$ $= \frac{1}{6}(2m)(2m+1)(4m+1)$ $+ 2\times\frac{1}{2}(2m)(2m+1) - 6m$	**M1 A1**	**M1** Expands and uses the standard results for $\sum r^2$ and $\sum r$ with $n=2m$. Condone slips in the constant term. **A1** Correct expression, oe, unsimplified.
	$= \frac{1}{3}m\left[(8m^2+6m+1)+6(2m+1)-18\right]$	**dM1**	Takes out $\frac{1}{3}m$ and expands to a quadratic in the bracket. Condone arithmetic slips.
	$= \frac{1}{3}m(8m^2+18m-11)$ $a=8,\ b=18,\ c=-11$	**A1**	Expression in the required form, or the three values stated.
		[4]	
9(b)	$\sum_{r=1}^{4m} = \frac{1}{3}(2m)\left(8(2m)^2+18(2m)-11\right)$	**M1**	Replaces m by $2m$ in their (a) result of the form $\frac{1}{3}m(am^2+bm+c)$, or applies the standard results with $n=4m$.
	$\sum_{r=2m+1}^{4m} =$ "$\frac{1}{3}m(64m^2+72m-22)$" $-$ "$\frac{1}{3}m(8m^2+18m-11)$"	**dM1**	Subtracts their sum to $2m$ from their sum to $4m$.
	$= \frac{1}{3}m(56m^2+54m-11)$	**A1 AG**	Correct expressions for both sums, subtracted to reach the printed result without error.
	$2m+1=21 \Rightarrow m=10$	**B1**	
	$\frac{10}{3}(5600+540-11) = 20430$	**B1**	From substituting $m=10$ in the printed result.
		[5]	
	Total for Question 9	**9**	

Question	Scheme	Marks	Notes
10(i)	$r(r+1)(4r-k) = 4r^3 + (4-k)r^2 - kr$	**B1**	oe, e.g. $4r^3 + 4r^2 - kr^2 - kr$.
	$4 \cdot \frac{1}{4}n^2(n+1)^2 + (4-k) \cdot \frac{1}{6}n(n+1)(2n+1)$ $- k \cdot \frac{1}{2}n(n+1)$	**M1 A1**	**M1** Applies the standard results to their expansion in powers of r, with at least two formulas correct. Condone coefficient slips. **A1** Correct expression; need not be simplified.
	$= \frac{1}{6}n(n+1)\left[6n^2 + 14n - 2kn + 4 - 4k\right]$	**dM1**	Takes out $n(n+1)$, or a multiple, and collects the rest into a quadratic in n; or expands both sides fully to compare. Condone slips.
	$= \frac{1}{3}n(n+1)(n+2)(3n+1-k)$	**A1 AG**	cso. Quadratic shown equal to $(n+2)(3n+1-k)$, by factorising or expanding the product, or full expansions shown equal. Working back from the printed result scores A0.
		[5]	
10(ii)	$\frac{1}{3}(2n)(2n+1)(2n+2)(6n) - \frac{1}{3}n(n+1)(n+2)(3n)$	**M1**	Uses $k = 1$ and subtracts the sum to n from the result of (i) with n replaced by $2n$. Not for subtracting the sum to $n+1$.
	$= n^2(n+1)\left[8(2n+1) - (n+2)\right]$	**dM1**	Takes out a common factor including $n^2(n+1)$, or expands and factorises with at least that factor. Condone slips in the bracket.
	$= 3n^2(n+1)(5n+2)$	**A1**	cao. $n^2(n+1)(15n+6)$ is not fully factorised and scores A0.
		[3]	
	Total for Question 10	**8**	

Question	Scheme	Marks	Notes
11(a)	$(2r-1)^3 = 8r^3 - 12r^2 + 6r - 1$ $\sum = 8\sum r^3 - 12\sum r^2 + 6\sum r - \sum 1$	**M1**	Expands the cube to four terms and writes the sum as separate sums. Condone one coefficient slip.
	$= 8 \times \frac{1}{4}n^2(n+1)^2 - 12 \times \frac{1}{6}n(n+1)(2n+1)$ $+ 6 \times \frac{1}{2}n(n+1) - n$	**M1 A1**	**M1** Substitutes the standard results for every sum in their expansion. Condone $\sum 1$ taken as 1, or one slip in a formula. **A1** Correct expression, unsimplified.
	$= (2n^4 + 4n^3 + 2n^2) - (4n^3 + 6n^2 + 2n)$ $+ (3n^2 + 3n) - n$	**dM1**	Expands the brackets and collects like terms to a polynomial in n. Condone sign slips.
	$= 2n^4 - n^2 = n^2(2n^2 - 1)$	**A1 AG**	Correct substituted expression leading without error to $n^2(2n^2-1)$; combined expansion steps accepted.
		[5]	
11(b)	$\sum_{r=n+1}^{2n} = (2n)^2\left(2(2n)^2 - 1\right) - n^2(2n^2-1)$	**M1**	Replaces n by $2n$ in the result of (a) and subtracts the result of (a) for n. Not for subtracting the sum to $n+1$.
	$= 30n^4 - 3n^2$	**A1**	
	$\sum_{r=1}^{\infty} 240\left(\frac{3}{5}\right)^r = \dfrac{144}{1-\frac{3}{5}} = 360$	**B1**	
	$30n^4 - 3n^2 = 29n^4 - 34n^2 + 360$ $\Rightarrow n^4 + 31n^2 - 360 = 0 \Rightarrow n^2 = \ldots$	**M1**	Equates their LHS of the form $an^4 + bn^2$ to the RHS with their sum to infinity, collects to three terms and solves as a quadratic in n^2; calculator allowed.
	$n^2 = 9$ $(n^2 \neq -40) \Rightarrow n = 3$	**A1**	$n = 3$ only; $n = -3$ and $n^2 = -40$ rejected or not given.
		[5]	
	Total for Question 11	**10**	

Question	Scheme	Marks	Notes
12(a)	$r(r+1)(2r+1) = 2r^3 + 3r^2 + r$	**B1**	
	$2 \times \frac{1}{4}n^2(n+1)^2 + 3 \times \frac{1}{6}n(n+1)(2n+1)$ $+ \frac{1}{2}n(n+1)$	**M1 A1**	**M1** Replaces each sum in their expansion by its standard result, with at least two results correct. Condone coefficient slips.
	$\frac{1}{2}n(n+1)(n^2+3n+2)$ $= \frac{1}{2}n(n+1)(n+1)(n+2)$	**dM1**	Takes out $\frac{1}{2}n(n+1)$ and factorises their quadratic, or expands fully and compares coefficients with $\frac{1}{2}n(n+a)^2(n+b)$.
	$\frac{1}{2}n(n+1)^2(n+2)$: $a = 1$, $b = 2$	**A1**	cso. Values stated or the factorised form shown.
		[5]	
12(b)	$\frac{1}{2}n(n+1)^2(n+2) - \frac{1}{2} \times 10 \times 11^2 \times 12$ $= 17\,940$	**M1**	Subtracts their (a) result at $n = 10$ from their result in n and equates to 17 940.
	$n(n+1)^2(n+2) = 50\,400$	**A1**	oe, e.g. $\frac{1}{2}n(n+1)^2(n+2) = 25\,200$ or an expanded quartic.
	$\left((n+1)^2 - 1\right)(n+1)^2 = 50\,400$ $\Rightarrow (n+1)^2 = \ldots$	**dM1**	Forms a three-term quadratic in $(n+1)^2$, or another valid substitution, and solves it by factorising, formula or completing the square. Not for trial or a calculator-solved quartic.
	$(n+1)^2 = 225 \Rightarrow n = 14$	**A1**	cso. $n = 14$ only, other roots rejected or not given. Without the algebraic method, at most M1 A1.
		[4]	
Total for Question 12		**9**	

Question	Scheme	Marks	Notes
13(a)	$\sum_{r=1}^{n}\left[(2r-1)^3-(2r)^3\right]$	**M1**	Writes the LHS as a sum over r of an odd cube minus the next even cube, oe; or as $\sum_{r=1}^{2n} r^3 - 16\sum_{r=1}^{n} r^3$.
	$(2r-1)^3-(2r)^3=-12r^2+6r-1$	**A1**	On the $\sum r^3$ route, for $n^2(2n+1)^2-4n^2(n+1)^2$ oe.
	$-12\times\frac{1}{6}n(n+1)(2n+1)+6\times\frac{1}{2}n(n+1)-n$	**M1**	Substitutes the standard results for $\sum r^2$ and $\sum r$ into their quadratic general term, or for $\sum r^3$ at $2n$ and n. Condone coefficient slips.
	$=n\left[-2\left(2n^2+3n+1\right)+3n+3-1\right]$	**dM1**	Takes out a factor of n, or expands fully, and collects terms towards a cubic in n.
	$=n\left(-4n^2-3n\right)=-n^2(4n+3)$	**A1 AG**	Correct substituted expression leading without error to the printed result; combined steps accepted.
		[5]	
13(b)	Smallest $2m+1$: $-(m+5)^2(4m+23)+m^2(4m+3)=-3725$	**M1 A1**	**M1** Takes the smallest as $2m+1$ oe, in any letter, and subtracts the result of (a) at $n=m$ from that at $n=m+5$, oe. Condone a sign slip. **A1** Correct equation, oe.
	$2m^2+11m-105=0\Rightarrow m=\ldots$	**dM1**	Expands to a three-term quadratic and solves it by factorising, formula or completing the square; a calculator is allowed.
	$m=5\Rightarrow$ smallest integer 11	**A1**	cso, with the negative root rejected or ignored. 11 by trial, without using part (a), scores nothing.
		[4]	
	Total for Question 13	**9**	

Question	Scheme	Marks	Notes
14(a)	$6 \cdot \frac{1}{6}n(n+1)(2n+1) + 2 \cdot \frac{1}{2}n(n+1) - 8n$	**M1 A1**	**M1** Uses both standard formulae and replaces $\sum 8$ by $8n$. Condone a slip in one coefficient. **A1** Correct expression, simplified or not.
	$= 2n\left[(n+1)^2 - 4\right]$ $= 2n(n-1)(n+3)$	**dM1**	Takes out a factor n and factorises the remaining quadratic into linear factors, by difference of two squares or any method.
	$a = 2,\ b = 1,\ c = 3$	**A1**	cso. Accept $2n(n-1)(n+3)$ as the final form.
		[4]	
14(b)	$\sin(90r)^\circ = 1, 0, -1, 0$ for $r = 1, 2, 3, 4$, repeating every four terms	**B1**	Pattern and its repetition; may be implied by $1 + 0 - 3 + 0 + 5 + 0 - 7 + \cdots$.
	$(4m+1) - (4m+3) = -2$ per block of four $\Rightarrow$ first $4k$ terms: $-2k$; last term $4k+1$	**M1**	Splits the terms into repeating blocks or pairs, finds a general block total, counts the blocks and accounts for any leftover term.
	$-2k + (4k+1) = 2k+1$	**A1 AG**	Final term shown to be $4k+1$; printed result reached without error from a general block or pair sum. Checks for particular k earn nothing.
		[3]	
14(c)	$2n(n-1)(n+3) - (6+2-8) - (24+4-8)$	**M1**	Subtracts the $r = 1$ and $r = 2$ terms, or their (a) expression at $n = 2$, from their (a) expression.
	$= 2n(n-1)(n+3) - 20$	**A1**	oe, such as $2n^3 + 4n^2 - 6n - 20$.
	$k = n$: $(2n+1) - 1\sin 90^\circ - 2\sin 180^\circ$	**M1**	Uses (b) with $k = n$ and subtracts the $r = 1$ and $r = 2$ terms.
	$= 2n \Rightarrow \text{RHS} = 3(2n)^2 = 12n^2$	**A1**	
	$2n(n-1)(n+3) - 20 = 12n^2$ $\Rightarrow n^3 - 4n^2 - 3n - 10 = 0$ $(n-5)(n^2+n+2) = 0$	**M1**	Equates their sides to form a cubic and attempts to solve it, with a root by trial, the factor theorem or a calculator, then a quadratic factor. Condone arithmetic slips.

Question 14 continued

Question	Scheme	Marks	Notes
	$n = 5$ $n^2 + n + 2$: $1^2 - 4(1)(2) = -7 < 0$, no real roots	**A1**	cso. Needs the discriminant, or $(n + \frac{1}{2})^2 + \frac{7}{4} > 0$, or $n^2 + n + 2 > 0$ for $n > 0$, with a conclusion of no other roots. A calculator statement alone is not enough.
		[6]	
	Total for Question 14	**13**	

Question	Scheme	Marks	Notes
15(a)	$(-1)^r = -1$ for odd r, $(-1)^r = 1$ for even r	**B1**	
	Pairs $r = 2k-1$ and $r = 2k$, $k = 1, \ldots, n$: $(-1)^{2k-1} f(2k-1) + (-1)^{2k} f(2k)$ $= f(2k) - f(2k-1)$	**E1**	Pairs each odd term with the next even term, giving n pairs each $f(2r) - f(2r-1)$, summed for $r = 1$ to n. Condone not renaming the index.
		[2]	
15(b)	$f(2r) - f(2r-1) = (2r)^2(2r+1) - (2r-1)^2(2r)$	**M1**	Substitutes $2r$ and $2r-1$ for r in $r^2(r+1)$, subtracts in that order and expands.
	$= 12r^2 - 2r$	**A1**	
	$\sum_{r=1}^{n} \left(12r^2 - 2r\right)$ $= 12 \times \frac{1}{6} n(n+1)(2n+1) - 2 \times \frac{1}{2} n(n+1)$	**M1**	Sums their $f(2r) - f(2r-1)$, a polynomial in r, from 1 to n, using the correct standard result for each power.
	$= n(n+1)(4n+2-1) = n(n+1)(4n+1)$	**A1 AG**	Correct substituted expression leading without error to the printed result; both sides expanded to $4n^3 + 5n^2 + n$ with a conclusion accepted.
		[4]	
15(c)	$(-1)^r r^2(r+1) + 3(r^2 + r)$ $3\sum_{r=1}^{2n}(r^2 + r)$ $= 3\left[\frac{1}{6}(2n)(2n+1)(4n+1) + \frac{1}{2}(2n)(2n+1)\right]$	**M1**	Splits the term into $(-1)^r r^2(r+1)$ and $3(r^2+r)$, applying both standard results with n replaced by $2n$.
	$= 4n(n+1)(2n+1)$	**A1**	oe, such as $8n^3 + 12n^2 + 4n$.
	$n(n+1)(4n+1) + 4n(n+1)(2n+1)$ $= n(n+1)(12n+5)$	**A1 AG**	Adds the part (b) result to a correct sum, reaching the printed result without error.
		[3]	
15(d)	Sum to $r = 40$, plus the $r = 41$ term, minus sum to $r = 10$, using part (c) with $n = 20$ and $n = 5$	**M1**	Subtracts the sum to $r = 10$ from the sum to $r = 41$, found as sum to 40 plus the $r = 41$ term, or sum to 42 minus the $r = 42$ term.
	$20 \times 21 \times 245 = 102\,900$, $5 \times 6 \times 65 = 1950$	**A1**	Or sum to 42 $= 118\,734$ with 1950.
	$41 \times 42 \times (-41 + 3) = -65\,436$	**B1**	Or the $r = 42$ term, 81 270, on the sum-to-42 route.

Question 15 continued

Question	Scheme	Marks	Notes
	$102\,900 - 65\,436 - 1950 = 35\,514$	**A1**	cao
		[4]	
	Total for Question 15	**13**	