CORE PURE / MISCELLANEOUS PURE

Roots of Polynomials

Mark scheme 15 questions · 115 marks

EXAM BOARDS Edexcel 9FM0 · AQA 7367 · OCR A H245 · OCR MEI H645 · CIE 9231

Questions

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	Total	**115**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [***n***] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$p = -6$	**B1**	
		[1]	
1(b)	$\alpha\beta + \beta\gamma + \gamma\alpha = -4$, $\alpha\beta\gamma = \frac{8}{3}$	**B1**	Both values needed.
		[1]	
1(c)	$\frac{(\alpha+1)(\beta+1)(\gamma+1)}{\alpha\beta\gamma}$ $= \frac{\alpha\beta\gamma+(\alpha\beta+\beta\gamma+\gamma\alpha)+(\alpha+\beta+\gamma)+1}{\alpha\beta\gamma}$	**M1 A1**	**M1** Writes the product over $\alpha\beta\gamma$, or expands it directly, giving terms in the root sums. Condone one slip in the expansion. **A1** Correct expression in $\alpha+\beta+\gamma$, $\alpha\beta+\beta\gamma+\gamma\alpha$ and $\alpha\beta\gamma$, oe such as $1+\sum\frac{1}{\alpha}+\sum\frac{1}{\alpha\beta}+\frac{1}{\alpha\beta\gamma}$ rewritten in them.
	$= \frac{\frac{8}{3}-4+2+1}{\frac{8}{3}} = \frac{5}{8}$	**A1**	$\frac{5}{8}$ oe, such as 0.625.
		[3]	
	Total for Question 1	**5**	

Question	Scheme	Marks	Notes
2(a)	$\alpha+\beta+\gamma=-5,\ \alpha\beta+\beta\gamma+\gamma\alpha=-4,\ \alpha\beta\gamma=24$	**B1**	
		[1]	
2(b)(i)	$\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}=\frac{\text{“}-5\text{”}}{\text{“}24\text{”}}$	**M1**	Combines over the common denominator $\alpha\beta\gamma$ to the sum of roots over the product and substitutes their values from (a).
	$-\frac{5}{24}$	**A1**	oe, or awrt -0.208.
		[2]	
2(b)(ii)	$\alpha\beta\gamma+2(\alpha\beta+\beta\gamma+\gamma\alpha)+4(\alpha+\beta+\gamma)+8$ $=\text{“}24\text{”}+2(\text{“}-4\text{”})+4(\text{“}-5\text{”})+8$	**M1**	Expands to the three root sums and a constant, or uses $x=y-2$, and substitutes their values. Condone one coefficient slip.
	4	**A1**	
		[2]	
2(b)(iii)	$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2$ $=(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)$	**M1**	Writes the sum as $(\alpha\beta+\beta\gamma+\gamma\alpha)^2$ minus a multiple of $\alpha\beta\gamma(\alpha+\beta+\gamma)$. Condone a wrong coefficient.
	$=(-4)^2-2(24)(-5)$	**A1**	Correct identity with the correct values substituted.
	$=256$	**A1**	
		[3]	
	Total for Question 2	**8**	

Question	Scheme	Marks	Notes
3(a)	$\alpha+\beta+\gamma=-2,\ \alpha\beta+\beta\gamma+\gamma\alpha=-5,\ \alpha\beta\gamma=3$	**B1**	All three, soi.
	$(\alpha\beta+\beta\gamma+\gamma\alpha)^2$ $=\sum\alpha^2\beta^2+2(\alpha\beta^2\gamma+\beta\gamma^2\alpha+\gamma\alpha^2\beta)$	**M1**	Squares $\sum\alpha\beta$, giving the three squared terms and cross terms. Condone a slip in the cross terms' coefficient.
	$\sum\alpha^2\beta^2=\left(\sum\alpha\beta\right)^2-2\alpha\beta\gamma\sum\alpha$	**A1**	Correct identity in terms of $\sum\alpha\beta$, $\alpha\beta\gamma$ and $\sum\alpha$, oe.
	$=(-5)^2-2(3)(-2)=37$	**A1**	cso: withhold if sign errors in the root sums cancel.
		[4]	
3(b)	$\sum\alpha^2=(-2)^2-2(-5)\ [=14]$	**M1**	Uses $\left(\sum\alpha\right)^2-2\sum\alpha\beta$ with their values. Condone sign slips in substitution.
	$\alpha^2\beta^2\gamma^2=3^2=9$	**B1**	
	$x^3-14x^2+37x-9=0$	**A1**	Or any integer multiple, in any variable; must be an equation.
		[3]	
	Total for Question 3	**7**	

Question	Scheme	Marks	Notes
4	$(a-d)+a+(a+d)=-3 \Rightarrow a=\ldots$	**M1**	Equates the sum of equally spaced roots, such as $a-d,\ a,\ a+d$ or $a,\ a+d,\ a+2d$, to ± 3.
	$a=-1$	**A1**	Middle root -1, in any parametrisation.
	$(a-d)a(a+d)=20$	**M1 A1**	**M1** Equates the product of their three roots to ± 20. **A1** Correct equation, oe such as $a(a^2-d^2)=20$ or with $a=-1$ substituted.
	$-1\left(1-d^2\right)=20 \Rightarrow d^2=\ldots\ [=21]$	**dM1**	Substitutes their a and solves for d or d^2.
	$x=-1-\sqrt{21},\ -1,\ -1+\sqrt{21}$	**A1**	All three, exact or awrt -5.58 and 3.58, with no others.
	Total for Question 4	**6**	

Question	Scheme	Marks	Notes
5(a)	$\alpha + \beta = k, \quad \alpha\beta = 1$	**B1**	Both values needed.
		[1]	
5(b)	$\alpha^2 + \beta^2 + \alpha + \beta$ $= (\alpha + \beta)^2 - 2\alpha\beta + (\alpha + \beta)$	**M1**	Writes $\alpha^2 + \beta^2$ in terms of $\alpha + \beta$ and $\alpha\beta$ and substitutes their values from (a). Condone a sign slip in the identity.
	$= k^2 + k - 2$	**A1**	Must be simplified.
		[2]	
5(c)	$(\alpha^2 + \beta)(\beta^2 + \alpha)$ $= (\alpha\beta)^2 + \alpha^3 + \beta^3 + \alpha\beta$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	**M1**	Expands the product to four terms and expresses $\alpha^3 + \beta^3$ in terms of k by any valid identity or reduction. Condone sign and coefficient slips.
	Product $= k^3 - 3k + 2$	**A1**	
	$x^2 - (k^2 + k - 2)x + k^3 - 3k + 2 = 0$	**A1**	Or any integer multiple, in x, with $= 0$; brackets need not be expanded. Stating p, q, r alone scores A0.
		[3]	
Total for Question 5		**6**	

Question	Scheme	Marks	Notes
6(a)	$(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2$ $=2\sum\alpha^2-2\sum\alpha\beta$ $=2\big((\sum\alpha)^2-2\sum\alpha\beta\big)-2\sum\alpha\beta$	**M1**	Expands the squares and writes the expression in terms of $\sum\alpha$ and $\sum\alpha\beta$ only. Condone sign and coefficient slips.
	$=2\big((\sum\alpha)^2-3\sum\alpha\beta\big)$	**A1**	oe, such as $2(\sum\alpha)^2-6\sum\alpha\beta$.
	$\sum\alpha=5,\ \sum\alpha\beta=p$ $14=2(5^2-3p)\Rightarrow p=6$	**A1 AG**	Correct $\sum\alpha=5$ and $\sum\alpha\beta=p$ substituted into a correct expression, leading without error to $p=6$.
		[3]	
6(b)	$\alpha\beta\gamma=q$	**B1**	
	$\frac{\sum\alpha\beta}{\alpha\beta\gamma}=\frac{6}{q}=3$	**M1**	Writes the reciprocal sum as $\sum\alpha\beta$ over $\alpha\beta\gamma$, substitutes their product and p, and sets it equal to 3. Condone a sign slip in the product.
	$q=2$	**A1**	
		[3]	
6(c)	$(\alpha+1)(\beta+1)(\gamma+1)$ $=\alpha\beta\gamma+\sum\alpha\beta+\sum\alpha+1$ $=$ "2" $+6+5+1$	**M1**	Expands the product into $\alpha\beta\gamma$, $\sum\alpha\beta$, $\sum\alpha$ and 1, condoning one slip, and substitutes their values; or any method not finding the roots, eg $-f(-1)$.
	$=14$	**A1**	cao. Scores 0 if the roots are found or no working is shown.
		[2]	
	Total for Question 6	**8**	

Question	Scheme	Marks	Notes
7	$\alpha+\beta+\gamma=4,\ \sum\alpha\beta=-1,\ \alpha\beta\gamma=-6$	**B1**	All three correct, soi.
	Single fraction, then root sums: $\dfrac{\sum(\beta+\gamma)(\gamma+\alpha)}{(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)}=\ldots$	**M1**	Forms a single fraction and attempts numerator and denominator in $\sum\alpha$, $\sum\alpha\beta$, $\alpha\beta\gamma$, or via $\alpha+\beta=4-\gamma$ oe. Condone expansion slips.
	$=\dfrac{(\sum\alpha)^2+\sum\alpha\beta}{\sum\alpha\sum\alpha\beta-\alpha\beta\gamma}$	**A1**	Correct numerator and denominator, oe, or values 15 and 2 found.
	$=\dfrac{16-1}{-4+6}=\dfrac{15}{2}$	**A1**	$\frac{15}{2}$ oe.
	Total for Question 7	**4**	

Question	Scheme	Marks	Notes
8(a)	$y = \frac{1}{x+1} \Rightarrow x = \frac{1-y}{y}$	**B1**	oe, such as $x = \frac{1}{y} - 1$.
	$\left(\frac{1-y}{y}\right)^3 + \left(\frac{1-y}{y}\right)^2 - 2\left(\frac{1-y}{y}\right) - 1 = 0$	**M1**	Substitutes their x in terms of y into every term of the given cubic.
	$(1-y)^3 + y(1-y)^2 - 2y^2(1-y) - y^3 = 0$ $\Rightarrow \ldots$	**M1**	Clears the fractions and expands to a polynomial in y. Condone slips in expansion.
	$y^3 - y^2 - 2y + 1 = 0$	**A1**	Or any multiple, as an equation in any variable.
		[4]	
8(b)	Sum of roots $= 1$, sum of pairs $= -2$	**B1**	Both values.
	"1"$^2 - 2($"-2"$) = \ldots$	**M1**	Uses (sum of roots)$^2 - 2$(sum of pairs) with their values.
	$= 5$	**A1**	With no incorrect values seen.
		[3]	
8(c)	Product of roots $= -1$	**B1**	
	"1"$^3 - 3($"1"$)($"-2"$) + 3($"-1"$) = \ldots$	**M1**	Uses a correct identity for the sum of cubes, or sums the cubic over its roots, with their values. Not for the identity alone.
	$= 4$	**A1**	
		[3]	
	Total for Question 8	**10**	

Question	Scheme	Marks	Notes
9(a)	$\frac{a}{r} + a + ar = 14$	**M1 A1**	**M1** Writes the roots as a geometric progression, such as $\frac{a}{r}, a, ar$ or a, ar, ar^2, in the sum of roots. Condone a sign error in 14. **A1** Correct sum relation, oe.
	$\frac{a^2}{r} + a^2 + a^2 r = 56$ $\Rightarrow a^2\left(\frac{1}{r} + 1 + r\right) = 56 \Rightarrow a = \ldots$	**M1**	Forms the sum of products in pairs equal to 56 in their progression and eliminates r, e.g. by dividing by the sum relation. Condone slips.
	$a = 4$	**A1**	Middle root 4, e.g. $ar = 4$ for roots a, ar, ar^2.
	"4" $\left(\frac{1}{r} + 1 + r\right) = 14$ $\Rightarrow 2r^2 - 5r + 2 = 0 \Rightarrow r = \ldots$ $[= \frac{1}{2}, 2]$	**dM1**	Substitutes their middle root into a relation to form a three-term quadratic in r and solves it by any means, leading to $r = \ldots$
	Roots 2, 4, 8	**A1**	All three roots stated; values of r alone do not score.
		[6]	
9(b)	$c = -(2 \times 4 \times 8) = -64$	**B1ft**	ft minus the product of their three roots.
		[1]	
	Total for Question 9	**7**	

Question	Scheme	Marks	Notes
10(a)	$\sum \alpha\beta\gamma = 4, \quad \alpha\beta\gamma\delta = 1$	**B1**	
	$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{\sum \alpha\beta\gamma}{\alpha\beta\gamma\delta} = \ldots$	**M1**	Writes the reciprocal sum as their triple-product sum over their product of roots, or uses the sum of roots of the reciprocal equation.
	$= 4$	**A1**	
		[3]	
10(b)	$\sum \frac{1}{\alpha\beta} = \frac{\sum \alpha\beta}{\alpha\beta\gamma\delta} = 3$	**B1**	
	$\sum \frac{1}{\alpha^2} = \left(\sum \frac{1}{\alpha}\right)^2 - 2\sum \frac{1}{\alpha\beta}$ $= \text{“4”}^2 - 2(\text{“3”}) = \ldots$	**M1**	Uses the square of their reciprocal sum minus twice their pair sum of reciprocals.
	$= 10$	**A1**	
		[3]	
10(c)	$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = 4 - \frac{1}{\delta}$ $\frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = 4 - \frac{1}{\alpha}$ $\frac{1}{\gamma} + \frac{1}{\delta} + \frac{1}{\alpha} = 4 - \frac{1}{\beta}$ $\frac{1}{\delta} + \frac{1}{\alpha} + \frac{1}{\beta} = 4 - \frac{1}{\gamma}$	**M1 A1**	**M1** Writes at least one bracket as the sum of all four reciprocals minus the omitted reciprocal, or expands one bracket squared into its three squares and twice its pair products. **A1** All four correct, or that bracket's expansion correct.
	$\sum \left(4 - \frac{1}{\alpha}\right)^2 = \sum \left(16 - \frac{8}{\alpha} + \frac{1}{\alpha^2}\right)$ $= 64 - 8\sum \frac{1}{\alpha} + \sum \frac{1}{\alpha^2}$	**M1 A1**	**M1** Expands the squared brackets and sums, collecting in terms of $\sum \frac{1}{\alpha}$ and $\sum \frac{1}{\alpha^2}$, or of $\sum \frac{1}{\alpha^2}$ and $\sum \frac{1}{\alpha\beta}$. Condone coefficient slips. **A1** Correct collected form, oe such as $3\sum \frac{1}{\alpha^2} + 4\sum \frac{1}{\alpha\beta}$.
	$= 64 - 32 + 10 = 42$	**A1**	
		[5]	
	Total for Question 10	**11**	

Question	Scheme	Marks	Notes
11(a)	$t = \frac{1}{x-1} \Rightarrow x = 1 + \frac{1}{t}$	**B1**	Or $x = \frac{1+t}{t}$ oe; any letter.
	$\left(1+\frac{1}{t}\right)^3 - 3\left(1+\frac{1}{t}\right) + 1 = 0$	**M1**	Substitutes their x in terms of t into the cubic.
	$\frac{1}{t^3} + \frac{3}{t^2} - 1 = 0 \Rightarrow 1 + 3t - t^3 = 0$	**M1**	Expands the cube and multiplies through by t^3 to reach a polynomial equation. Condone slips in the expansion.
	$t^3 - 3t - 1 = 0$	**A1**	Or any multiple, in any variable; must be an equation.
		[4]	
11(b)	$\sum r = 0,\ \sum r_1 r_2 = -3$	**B1**	Both values, from the cubic in (a).
	$\sum r^2 = \text{“0”}^2 - 2(\text{“}-3\text{”})$	**M1**	Uses $(\sum r)^2 - 2\sum r_1 r_2$ with their values from a cubic in the reciprocal variable.
	$= 6$	**A1**	
		[3]	
11(c)	$r^3 = 3r + 1 \Rightarrow r^5 = 3r^3 + r^2$ $= r^2 + 9r + 3$	**M1 A1**	**M1** Uses $r^3 = 3r + 1$ to express r^5 in powers below 3 and sums over the roots, or uses the recurrence $\sum r^{n+3} = 3\sum r^{n+1} + \sum r^n$. Condone slips. **A1** oe, such as $\sum r^5 = 3\sum r^3 + \sum r^2$ with $\sum r^3 = 3$.
	$\sum r^5 = \text{“6”} + 9(0) + 3 \times 3 = 15$	**A1**	
		[3]	
	Total for Question 11	**10**	

Question	Scheme	Marks	Notes
12(a)	$x = \frac{1}{y}$	**B1**	Or $y = \frac{1}{x}$; any letter.
	$\frac{3}{y^4} - \frac{1}{y^3} - \frac{10}{y^2} + \frac{8}{y} + 4 = 0$ $\Rightarrow 3 - y - 10y^2 + 8y^3 + 4y^4 = 0$	**M1**	Substitutes a reciprocal into every term and multiplies through by a power of y to clear fractions. Condone sign and coefficient slips.
	$4x^4 + 8x^3 - 10x^2 - x + 3 = 0$	**A1**	Or any non-zero integer multiple; any variable.
		[3]	
12(b)	$x = 1 + \frac{2}{y}$	**B1**	Or $y = \frac{2}{x-1}$, or $x = \frac{y+2}{y}$; any letter.
	$3\left(1+\frac{2}{y}\right)^4 - \left(1+\frac{2}{y}\right)^3 - 10\left(1+\frac{2}{y}\right)^2$ $+ 8\left(1+\frac{2}{y}\right) + 4 = 0$	**M1**	Substitutes their x of the form $1 + \frac{k}{y}$, oe, into every term of the quartic.
	$3(y+2)^4 - y(y+2)^3 - 10y^2(y+2)^2$ $+ 8y^3(y+2) + 4y^4 = 0 \Rightarrow \ldots$	**M1**	Expands every power, before or after multiplying through by y^4, and collects to a polynomial in y. Condone arithmetic slips.
	$2x^4 - 3x^3 + 10x^2 + 44x + 24 = 0$	**A1**	Or any non-zero integer multiple, such as $4x^4 - 6x^3 + 20x^2 + 88x + 48 = 0$; any variable.
		[4]	
	Total for Question 12	**7**	

Question	Scheme	Marks	Notes
13(a)	$\alpha+\beta+\gamma+\delta=-b \Rightarrow b=2$	**B1**	
	$\sum\frac{1}{\alpha}=\frac{\sum\alpha\beta\gamma}{\alpha\beta\gamma\delta}=\frac{-d}{3}$ $\Rightarrow \frac{-d}{3}=2 \Rightarrow d=\ldots$	**M1**	Writes the reciprocal sum as the sum of triple products over the product of roots, in terms of d and 3, and solves for d. Condone a sign slip.
	$d=-6$	**A1**	
	$\sum\alpha^2+2\sum\alpha+4=6$ $(\sum\alpha)^2=\sum\alpha^2+2\sum\alpha\beta$	**M1**	Expands the squared condition into $\sum\alpha^2$, $\sum\alpha$ and a constant, and relates $\sum\alpha^2$ to $(\sum\alpha)^2$ and c. Condone slips in coefficients or the constant.
	$\sum\alpha^2=6$	**A1**	soi, eg within the equation for c.
	$(-2)^2=6+2c \Rightarrow c=-1$	**A1**	
		[6]	
13(b)	$\sum\alpha^3+3\sum\alpha^2+3\sum\alpha+4=20$	**M1**	Expands the cubed condition into $\sum\alpha^3$, $\sum\alpha^2$, $\sum\alpha$ and a constant. Condone slips in the coefficients or the constant.
	$\sum\alpha^3+18-6+4=20$ $\Rightarrow \sum\alpha^3=4$	**A1**	
	$\sum\alpha^4=-2\sum\alpha^3+\sum\alpha^2+6\sum\alpha-12$ $=-2(\text{“}4\text{”})+6+6(-2)-12$	**M1**	Expresses $\sum\alpha^4$ in their lower power sums, by summing their quartic over the roots or otherwise, and substitutes. Condone a constant of -3 for -12.
	$\sum\alpha^4=-26$	**A1**	
		[4]	
	Total for Question 13	**10**	

Question	Scheme	Marks	Notes
14(a)	$y = x^2$	**B1**	States or uses $y = x^2$, or $x = \sqrt{y}$.
	$(x^2-1)^2 = -2x$ $\Rightarrow (y-1)^2 = -2\sqrt{y}$	**M1**	Substitutes $x = \sqrt{y}$ with the $\sqrt{y}$ term separated from the integer powers of y. Condone sign slips.
	$(y-1)^4 = 4y$ $\Rightarrow y^4 - 4y^3 + 6y^2 - 4y + 1 = 4y$	**M1**	Squares both sides to eliminate $\sqrt{y}$ and expands to a polynomial in y. Condone sign and coefficient slips.
	$y^4 - 4y^3 + 6y^2 - 8y + 1 = 0$	**A1**	Any non-zero multiple, in any variable; must be an equation.
	$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 4$	**B1ft**	ft minus the ratio of the cubic to quartic coefficients of their quartic; or 4 from $(\sum \alpha)^2 - 2\sum \alpha\beta$.
		[5]	
14(b)	$\sum \alpha^2 = 4,\ \sum \alpha^2\beta^2 = 6,\ \sum \alpha^2\beta^2\gamma^2 = 8$	**B1**	All three values, from the quartic in y or from the original roots.
	$\sum \alpha^6 = 4^3 - 3(4)(6) + 3(8)$	**M1**	Uses a correct identity for the sum of cubes in terms of the root sums, with their values; or any valid power-sum method.
	$= 16$	**A1**	
		[3]	
14(c)	$\sum \alpha^4 = 4^2 - 2(6)\ [= 4]$ $\sum \alpha^8 = 4\sum \alpha^6 - 6\sum \alpha^4 + 8\sum \alpha^2 - 4$ $= 4(\text{“16”}) - 6(4) + 8(4) - 4$	**M1**	Expresses $\sum \alpha^8$ in lower power sums via the quartic in y or another valid identity, finding $\sum \alpha^4$ and substituting their values. Condone slips.
	$= 68$	**A1**	
		[2]	
	Total for Question 14	**10**	

Question	Scheme	Marks	Notes
15(a)	$\left(z^2+\frac{1}{z^2}\right)-2\left(z+\frac{1}{z}\right)-1=0$ $\Rightarrow (w^2-2)-2w-1=0$	**M1**	Divides by z^2 and uses $z^2+\frac{1}{z^2}=w^2\pm 2$ to form an equation in w only. Condone sign and coefficient slips.
	$w^2-2w-3=0$	**A1**	
	Roots in reciprocal pairs $\Rightarrow w=3,3,-1,-1$ $(w^2-2w-3)^2=0$ $\Rightarrow w^4-4w^3-2w^2+12w+9=0$	**A1 AG**	States that roots occur in reciprocal pairs, or that each value of w arises twice, and reaches the printed quartic without error. A0 for squaring with no reason given.
		[3]	
15(b)	$z+\frac{1}{z}=3\Rightarrow z^2-3z+1=0$ $z+\frac{1}{z}=-1\Rightarrow z^2+z+1=0$ $\Rightarrow z=\ldots$	**M1**	Forms $z^2-wz+1=0$ for both values of w from the quartic and solves each by any means. Condone sign slips.
	$z=\frac{3\pm\sqrt{5}}{2}$	**A1**	Both, exact, oe.
	$z=\frac{-1\pm i\sqrt{3}}{2}$	**A1**	Both, exact, oe such as $e^{\pm 2\pi i/3}$.
		[3]	
Total for Question 15		**6**	