# Questions

**1.** Prove by induction that

$$n^5 - n$$

is divisible by 5 for all positive integers $n$ **[5]**

**2.** Prove by induction that

$$9^n - 8n - 1$$

is divisible by 64 for all positive integers $n \in \mathbb{Z}^+$ **[7]**

**3.** Prove by induction that, for $n \in \mathbb{Z}^+$,

$$13^n + 3 \cdot 4^n + 11$$

is divisible by 12 **[7]**

**4.** Prove by mathematical induction that, for all non-negative integers $n$,

$$2^n + 2 \cdot 3^{2n} + 4^{2n+1}$$

is divisible by 7 **[6]**

**5.** For each positive integer $n$, define

$$u_n = 5^{2n} - 2^n$$

Use proof by induction to show that 23 is a factor of $u_n$ for all $n \in \mathbb{Z}^+$ **[6]**

**6.** Prove by induction that, for $n \in \mathbb{Z}^+$,

$$7^n - 6n - 1$$

is divisible by 36 **[6]**

**7.** The function $f$ is defined by

$$f(n) = 9^n - 8n + 63$$

Prove by induction that $f(n)$ is divisible by 64 for $n \geq 1$ **[6]**

**8.** Prove by mathematical induction that

$$7^{2n} + 8 \times 13^n - 9$$

is divisible by 36 for all positive integers $n$ **[6]**