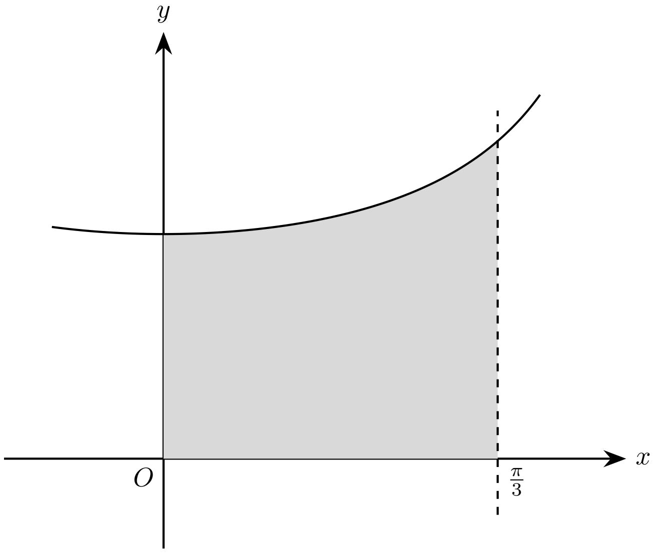
Questions

1. **(a)** Show that

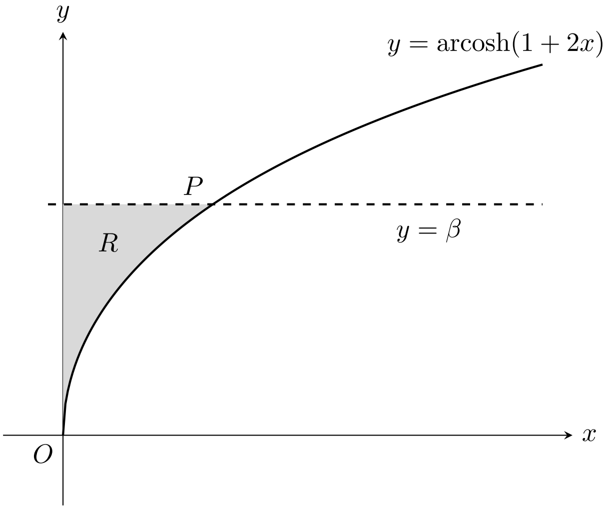
$$\frac{\mathrm{d}}{\mathrm{d}x}\ln(\sec x + \tan x) = \sec x$$ **[2]**



The shaded region is enclosed by the curve $y = \sqrt{\sec x}$, the coordinate axes and the line $x = \frac{\pi}{3}$.

The region is rotated completely about the x-axis.

(b) Find the exact volume of the solid generated. **[3]**



2. The diagram above shows a sketch of part of the curve with equation

$$y = \text{arcosh}(1 + 2x) \qquad x \geq 0$$

and the straight line with equation $y = \beta$.

The line and the curve intersect at the point with coordinates (α, β).

Given that $\beta = \ln(2 + \sqrt{3})$,

(a) show that $\alpha = \frac{1}{2}$. **[3]**

The finite region R, shown shaded in the diagram above, is bounded by the curve with equation $y = \text{arcosh}(1 + 2x)$, the y-axis and the line with equation $y = \beta$.

The region R is rotated through 2π radians about the y-axis.

(b) Use calculus to find the exact value of the volume of the solid generated. **[6]**

3. The equation of a curve is

$$y = \sqrt{\frac{k-x}{k+x}}$$

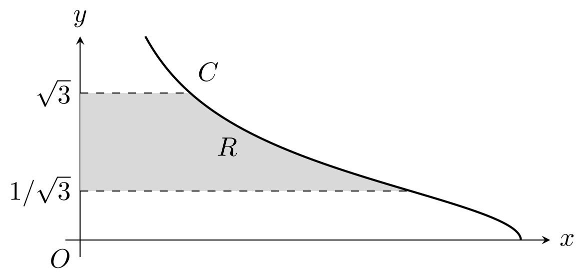
where k is a positive constant. The region enclosed by the curve, the x-axis, the y-axis and the line $x = k$ is rotated through 2π radians about the x-axis.

Given that the volume of the solid of revolution formed is 1 unit^3, find the exact value of k. **[4]**

4. **(a)** Find

$$\int \sin^2 \theta \, d\theta$$

[2]



The diagram shows part of the curve C with parametric equations $x = 6\sin^2\theta$, $y = \cot\theta$, $0 < \theta \leq \frac{\pi}{2}$.

The finite region R shown in the diagram is bounded by C, the lines $y = \dfrac{1}{\sqrt{3}}$, $y = \sqrt{3}$ and the y-axis. Region R is rotated through 2π radians about the y-axis to form a solid of revolution.

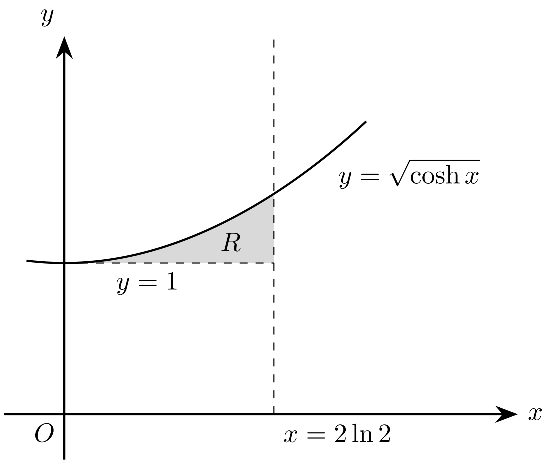
(b) Show that the volume of the solid of revolution formed is given by the integral

$$k \int_a^b \sin^2 \theta \, d\theta$$

where a, b and k are constants to be found. **[5]**

(c) Hence find the exact value of this volume, giving your answer in the form $p\pi^2$, where p is a constant to be found. **[3]**

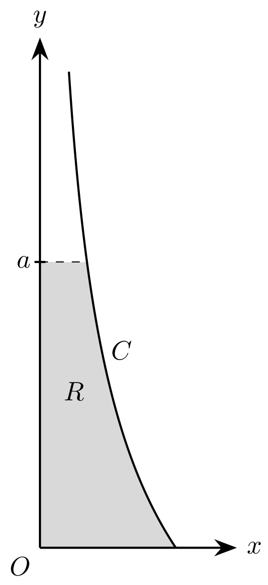
5. **(a)** Show that $\sinh(2\ln 2) = \dfrac{15}{8}$ **[2]**



The region R is bounded by the curve with equation $y = \sqrt{\cosh x}$, the line $y = 1$ and the line $x = 2\ln 2$, as shown in the diagram. The units of the axes are centimetres.

A designer models a hollow glass ornament as the solid formed by rotating R completely about the x-axis.

(b) Determine, according to the model, the exact volume of the ornament. **[4]**



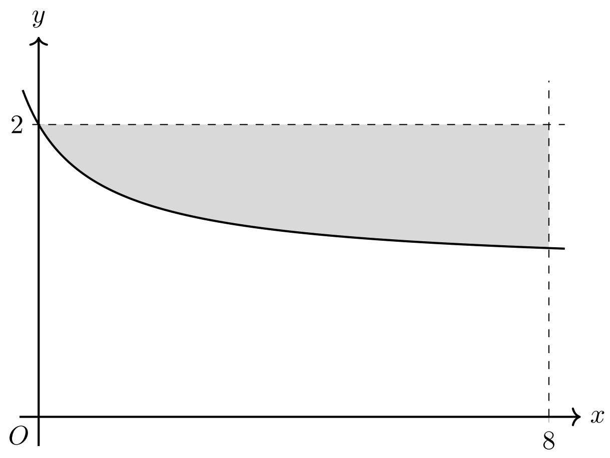
6. The curve C is given parametrically by

$$x = \frac{9}{t^4}, \qquad y = t^2 - 3, \qquad t \geq \sqrt{3}$$

The shaded region R, shown in the diagram, is enclosed by C, the x-axis, the y-axis and the line $y = a$. When R is rotated through 2π radians about the y-axis, the resulting solid has volume

$$\frac{7\pi}{8}$$

Find the exact value of a. **[8]**

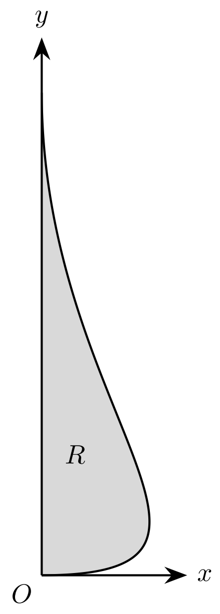


7. The diagram shows the region bounded by the curve $y = \sqrt{\dfrac{x+4}{x+1}}$, the line $y = 2$ and the line $x = 8$. The curve meets the line $y = 2$ at the point $(0, 2)$.

Find the exact volume of the solid formed when this region is rotated through 360° about the x-axis. **[6]**

8. The region in the first quadrant bounded by the curve $y = \sinh x$, the y-axis, and the line $y = 2$ is rotated through 360° about the x-axis.

Find the exact volume of revolution generated, expressing your answer in a form involving a logarithm. **[7]**

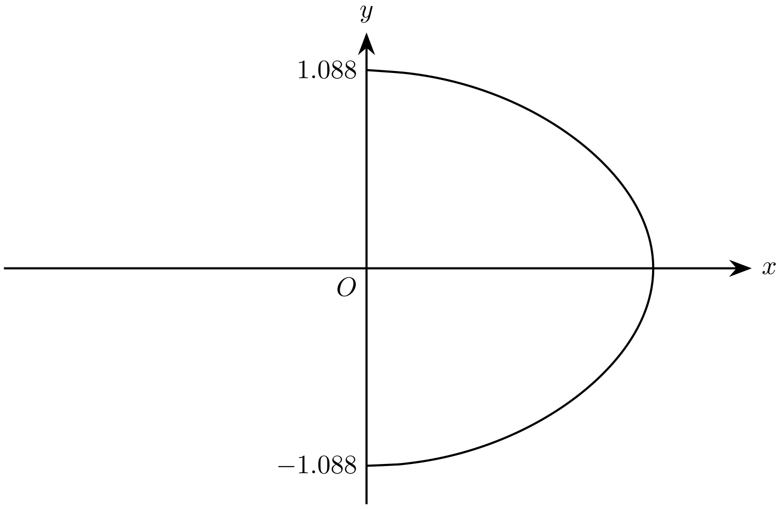


9. Part of the side profile of a solid wooden ornament is shown in the diagram.
The outline is modelled by the curve with parametric equations

$$x = 12t(1-t)^2, \quad y = 8t^2, \qquad 0 \le t \le 1$$

The ornament is formed by rotating the shaded region through 2π radians about the y-axis.

Use the model to find the volume of the ornament. **[7]**



10. A chocolatier makes hand-rolled chocolate drops.

The tallest drop in one tray was approximately 2.2 cm high.

The shape of this drop is modelled by rotating the curve with equation

$$20x^2 + 6y^2 + y\sin(2y) = 8, \qquad x \geq 0$$

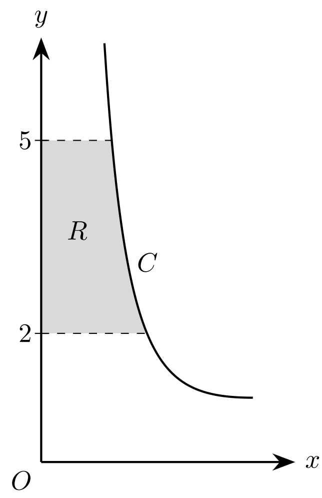
shown in the diagram above, about the y-axis through 2π radians, where the units are cm.

Given that the y-intercepts of the curve are -1.088 and 1.088 to four significant figures,

(a) Use algebraic integration to determine, according to the model, the volume of this chocolate drop. **[6]**

The chocolatier melts down 80 chocolate drops from the same tray.

(b) Use your answer to part (a) to decide whether, in reality, there is likely to be enough chocolate to fill a mould of volume $140\,\text{cm}^3$, giving a reason. **[2]**

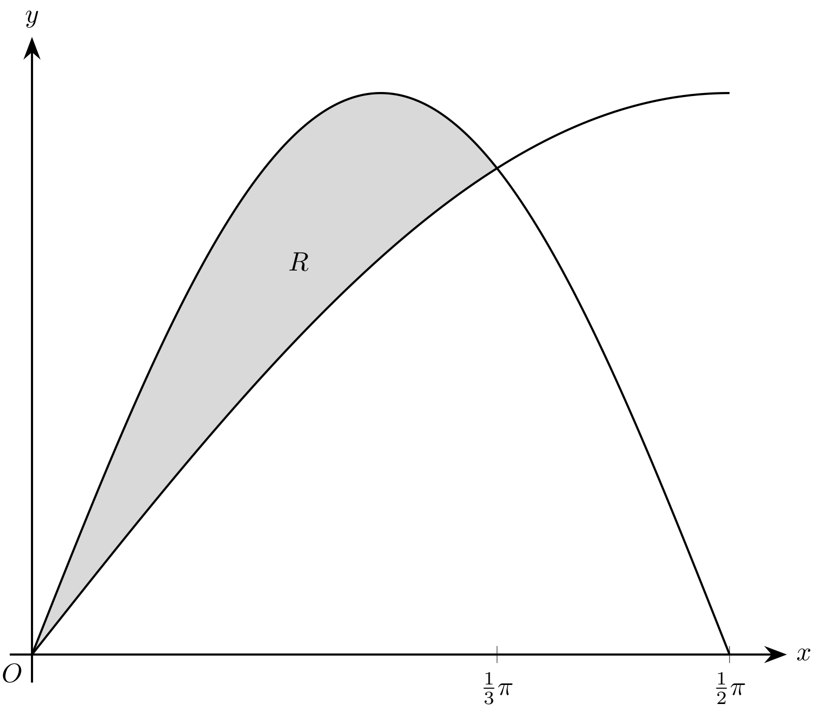


11. The diagram shows part of the curve C with parametric equations

$$x = \frac{2}{1+t}, \quad y = t^2 + 1, \qquad t \geq 0$$

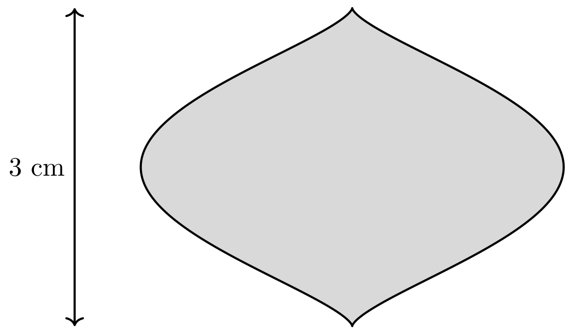
The region R is bounded by the curve, the y-axis and the lines $y = 2$ and $y = 5$. Region R is rotated through 2π radians about the y-axis.

Use parametric integration to find the volume of the resulting solid of revolution. **[6]**



12. The diagram shows the curves $y = \sin x$ and $y = \sin 2x$, for $0 \leq x \leq \frac{1}{2}\pi$. The shaded region R is the finite region enclosed by the curves.

Find the volume of the solid of revolution formed when R is rotated completely about the x-axis, giving your answer in terms of π. **[7]**



13. The diagram shows the outline of a glass bead. The bead is modelled by the solid obtained when the region enclosed by a curve C is rotated about the y-axis. The actual bead has height 3 cm. The curve C has parametric equations

$$x = \sin\theta - \frac{1}{3}\sin 3\theta, \qquad y = 1 - \cos\theta, \qquad 0 \le \theta \le 2\pi$$

(a) Show that a Cartesian equation of the curve C is

$$x^2 = \frac{16}{9}y^3(2-y)^3$$

[4]

(b) Hence, using the model, find, in cm^3, the volume of the bead. **[5]**

14. The equation of a curve is

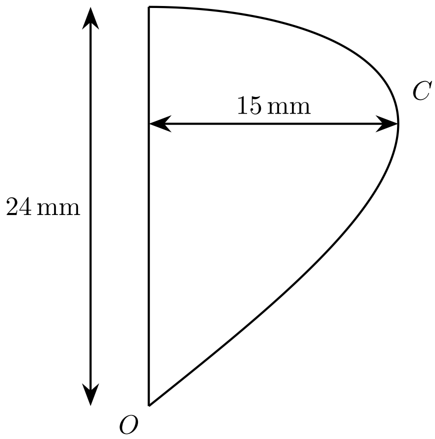
$$y = \frac{x}{\sqrt{k^2 + x^2}}$$

where k is a positive constant.

The region in the first quadrant bounded by the curve, the x-axis, the y-axis and the line $x = k$ is rotated through 2π radians about the x-axis
Given that the volume of revolution formed is 1 unit^3, find the exact value of k **[4]**

15. **(a)** Prove that

$$\cos 4\theta \cos \theta \equiv \frac{1}{2}(\cos 5\theta + \cos 3\theta)$$ **[3]**



The diagram shows the cross-section of a decorative ornament.

The ornament can be modelled by rotating the region enclosed by the curve C and the y-axis about the y-axis. Curve C has parametric equations

$$x = 15\sin 2\theta, \quad y = 24\sin\theta, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

where x and y are measured in millimetres.

(b) Find the volume of the ornament. **[5]**