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1. **(a)** Show that an appropriate integrating factor for

$$x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + y = 1, \qquad x > 0$$

is $e^{-1/x}$ **[4]**

(b) Hence find the solution of the differential equation

$$x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + y = 1, \qquad x > 0$$

for which $y = 3$ when $x = 1$ **[6]**

2. **(a)** Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\big(\ln(\cosh x + 1)\big) = \tanh\left(\frac{x}{2}\right)$$

[3]

(b) Hence solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \tanh\left(\frac{x}{2}\right) y = \sinh x$$

for which $y = 2$ when $x = 0$. Give your answer in exact form **[7]**

3. **(a)** For $x > -1$, determine the general solution of the differential equation

$$(1+x)\,\frac{\mathrm{d}y}{\mathrm{d}x} + y = \ln(1+x)$$

giving your answer in the form $y = f(x)$ **[4]**

(b) Given that $y = -1$ when $x = 0$, determine the positive value of x for which $y = 0$ **[2]**

4. (a) Show that, for $0 < x < \frac{\pi}{2}$, an appropriate integrating factor for

$$\cos x \frac{\mathrm{d}y}{\mathrm{d}x} + y \sin x = x$$

is $\sec x$ **[4]**

(b) Hence find the solution of

$$\cos x \frac{\mathrm{d}y}{\mathrm{d}x} + y \sin x = x$$

for $0 < x < \frac{\pi}{2}$, given that $y = 1$ when $x = \frac{\pi}{4}$ **[6]**

5. **(a)** Show that, for $x > 0$,

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\coth\frac{x}{2}\right)\right) = -\operatorname{cosech} x$$

[3]

(b) Hence solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \operatorname{cosech} x\, y = \tanh\frac{x}{2}, \qquad x > 0$$

for which $y = 1$ when $x = \ln 3$. Give your answer in exact form **[7]**

6. **(a)** Show that an appropriate integrating factor for

$$x(x-1)\frac{\mathrm{d}y}{\mathrm{d}x} + (x+1)y = x(x-1)^2$$

is $\dfrac{(x-1)^2}{x}$ **[4]**

(b) Hence find the solution of the differential equation

$$x(x-1)\frac{\mathrm{d}y}{\mathrm{d}x} + (x+1)y = x(x-1)^2$$

for $x > 1$, given that $y = 2$ when $x = 2$ **[6]**

7. Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \cot x\, y = \frac{\cos x}{\sin^2 x\left(1 + \ln^2(\sin x)\right)}$$

where $0 < x < \pi$ **[7]**

8. **(a)** Solve the differential equation

$$(1+x)\frac{\mathrm{d}y}{\mathrm{d}x} - 2y = (1+x)^4, \qquad x > -1$$

giving your answer in the form $y = f(x)$ **[3]**

(b) The solution curve passes through the point $(0, -1)$. Find the exact positive value of x for which $y = 0$ **[3]**

9. (a) Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\ln(\sec x + \tan x) = \sec x$$

[2]

(b) Hence find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - 2y\tan x = \sec^3 x$$

where $-\frac{\pi}{2} < x < \frac{\pi}{2}$. **[5]**

10. **(a)** Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\frac{\cosh x+1}{\sinh x}\right)\right)=-\operatorname{cosech} x$$

[3]

(b) Hence solve, for $x>0$,

$$\sinh x\,\frac{\mathrm{d}y}{\mathrm{d}x}-y=1-\cosh x$$

given that $y=2$ when $x=\ln 3$. Give your answer in exact form **[7]**

11. **(a)** Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2x}{x^2+1}y = 6x$$

giving your answer in the form $y = f(x)$ **[3]**

(b) Given that $y = \frac{7}{2}$ when $x = 0$, find the smallest positive value of x for which $y = 4$ **[3]**

12. For $x > 0$, solve the differential equation

$$\sinh x \frac{\mathrm{d}y}{\mathrm{d}x} - 2y \cosh x = \sinh^2 x \cosh x$$

given that $y = 5$ when $x = \ln(1 + \sqrt{2})$.
Give your answer in exact form. **[7]**

13. For $x > 0$, a function y satisfies

$$\sinh x \, \frac{\mathrm{d}y}{\mathrm{d}x} - y \cosh x = \sinh x \, \cosh^3 x$$

(a) By first writing the equation in the form $\frac{\mathrm{d}y}{\mathrm{d}x} + P(x)y = Q(x)$, find an integrating factor **[2]**

(b) Hence solve the differential equation, given that $y = 2$ when $x = \ln(1 + \sqrt{2})$ **[5]**

Give your answer in exact form.

14. For $x > 0$, solve the differential equation

$$\sinh x \frac{\mathrm{d}y}{\mathrm{d}x} - y \cosh x = \sinh x \cosh x$$

given that $y = 2$ when $x = \ln(1 + \sqrt{2})$.
Give your answer in exact form. **[7]**

15. Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - y\tan x = \frac{1}{\sqrt{1+\sin x}}$$

where $-\frac{\pi}{2} < x < \frac{\pi}{2}$ **[7]**