Questions

1. **(a)** Show that

$$3\cosh x - \sinh x = e^x + 2e^{-x}$$

[2]

(b) Hence solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} - 2y = 6(3\cosh x - \sinh x)$$

given that, when $x = 0$, $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$. **[10]**

2. Find the particular solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 4\frac{\mathrm{d}y}{\mathrm{d}t} + 13y = 8e^{-2t}\sin t$$

given that, when $t = 0$, $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 2$. **[10]**

3. A damped oscillator is governed by

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 6\frac{\mathrm{d}y}{\mathrm{d}t} + 13y = 75\sin 2t$$

(a) Find the general solution of this differential equation. **[6]**

(b) Show that, whatever the initial conditions, the motion is approximately of the form

$$y \approx R\sin(2t - \phi)$$

for large positive values of t, where $R > 0$ and $0 < \phi < \frac{\pi}{2}$. Determine R and ϕ. **[4]**

4. A function $y(t)$ satisfies the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y}{\mathrm{d}t} + 5y = 10\cos t$$

Given that, when $t = 0$, $y = 5$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 2$, find the particular solution. **[10]**

5. **(a)** Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 13y = 0$$

[2]

(b) Hence find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 13y = e^{-2x}(6\cos x - 8\sin x)$$

[4]

6. **(a)** Show that

$$\frac{\cosh x - \sinh x}{\cosh x + \sinh x} = e^{-2x}$$

[2]

(b) Hence find the particular solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = 4\left(\frac{\cosh x - \sinh x}{\cosh x + \sinh x}\right)$$

given that, when $x = 0$, $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$. **[10]**

7. Solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} - 6y = 50\sin x$$

subject to the conditions $y(0) = 3$ and $y'(0) = -6$. **[10]**

8. Determine the solution $y(x)$ of the initial value problem

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 4\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = xe^{2x}$$

subject to

$$y(0) = 1, \qquad \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0} = 0$$

[10]

9. **(a)** Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4y = 0$$

[2]

(b) Hence, by considering a particular integral of the form $x(A\cos 2x + B\sin 2x)$, find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4y = 8\cos 2x - 12\sin 2x$$

[4]

10. Find the particular solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 4\frac{\mathrm{d}y}{\mathrm{d}t} + 13y = 28\cos 3t - 4\sin 3t$$

given that, when $t = 0$, $y = 5$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 7$ **[10]**

11. Find the particular solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 4\frac{\mathrm{d}y}{\mathrm{d}t} + 5y = 8\cos t$$

given that, when $t = 0$, $y = 3$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 1$ **[10]**

12. **(a)** Show that, for all real x,

$$(1 + \tanh x)\cosh x = e^x$$ **[2]**

(b) Hence solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = 3(1 + \tanh x)\cosh x$$

given that, when $x = 0$, $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ **[10]**