Questions

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1. The infinite series C and S are defined by

$$C = \cos 2\theta - \frac{1}{3}\cos 5\theta + \frac{1}{9}\cos 8\theta - \frac{1}{27}\cos 11\theta + \cdots$$

$$S = \sin 2\theta - \frac{1}{3}\sin 5\theta + \frac{1}{9}\sin 8\theta - \frac{1}{27}\sin 11\theta + \cdots$$

Given that the series C and S are both convergent.

(a) Show that

$$C + \mathrm{i}S = \frac{3e^{2\mathrm{i}\theta}}{3 + e^{3\mathrm{i}\theta}} \qquad \textbf{[4]}$$

(b) Hence show that

$$S = \frac{9\sin 2\theta - 3\sin\theta}{10 + 6\cos 3\theta} \qquad \textbf{[4]}$$

2. (i) Show that

$$(1 - 2e^{2i\theta})(1 - 2e^{-2i\theta}) = 5 - 4\cos 2\theta$$ **[3]**

For a positive integer n, series C and S are defined by

$$C = \cos\theta + 2\cos 3\theta + 4\cos 5\theta + \cdots + 2^{n-1}\cos((2n-1)\theta)$$
$$S = \sin\theta + 2\sin 3\theta + 4\sin 5\theta + \cdots + 2^{n-1}\sin((2n-1)\theta)$$

(ii) Show that

$$C = \frac{2^{n+1}\cos((2n-1)\theta) - 2^n\cos((2n+1)\theta) - \cos\theta}{5 - 4\cos 2\theta}$$

and find a similar expression for S. **[9]**

3. **(i)** Show that

$$1 - e^{i2\theta} = 2\sin\theta(\sin\theta - i\cos\theta)$$

[3]

(ii) For a positive integer n, the series C and S are defined as follows.

$$C = 1 - \binom{n}{1}\cos 2\theta + \binom{n}{2}\cos 4\theta - \ldots + (-1)^n \cos 2n\theta$$

$$S = \quad -\binom{n}{1}\sin 2\theta + \binom{n}{2}\sin 4\theta - \ldots + (-1)^n \sin 2n\theta$$

By considering $C + iS$, show that

$$C = 2^n \sin^n\theta \cos\left(n\theta - \frac{n\pi}{2}\right)$$

and find a corresponding expression for S. **[9]**

4. The infinite series C and S are defined as follows:

$$C = \cos\alpha + r\cos(\alpha+\beta) + r^2\cos(\alpha+2\beta) + \ldots$$

$$S = \sin\alpha + r\sin(\alpha+\beta) + r^2\sin(\alpha+2\beta) + \ldots$$

where r is a real number and $|r| < 1$.

By considering $C + \mathrm{i}S$, show that

$$C = \frac{\cos\alpha - r\cos(\alpha-\beta)}{1 + r^2 - 2r\cos\beta}$$

and find a corresponding expression for S. **[8]**

5. (i) For values of θ for which the expressions are defined, show that

$$1 + \mathrm{i}\tan\theta = \sec\theta(\cos\theta + \mathrm{i}\sin\theta)$$ **[2]**

(ii) For a positive integer n, the series C and S are defined by

$$C = 1 - \binom{n}{2}\tan^2\theta + \binom{n}{4}\tan^4\theta - \cdots$$

$$S = \binom{n}{1}\tan\theta - \binom{n}{3}\tan^3\theta + \binom{n}{5}\tan^5\theta - \cdots$$

where in each case the pattern continues until the power of $\tan\theta$ is at most n.

By considering $C + \mathrm{i}S$, show that

$$C = \sec^n\theta\cos n\theta$$

and find a corresponding expression for S. **[7]**

6. The convergent series C and S are defined by

$$C = \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n \cos\big((4n+1)\theta\big)$$

$$S = \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n \sin\big((4n+1)\theta\big)$$

(a) Show that

$$C + \mathrm{i}S = \frac{2e^{\mathrm{i}\theta}}{2 + e^{4\mathrm{i}\theta}} \qquad \textbf{[4]}$$

(b) Hence show that

$$C = \frac{4\cos\theta + 2\cos 3\theta}{5 + 4\cos 4\theta} \qquad \textbf{[4]}$$

7. Let

$$C = \sum_{r=0}^{n-1} \cos^2\left(\theta + \frac{2\pi r}{n}\right)$$

$$S = \sum_{r=0}^{n-1} \sin\left(\theta + \frac{2\pi r}{n}\right) \cos\left(\theta + \frac{2\pi r}{n}\right)$$

where n is an integer greater than 2. Let $\exp(x)$ denote e^x.

By considering the real and imaginary parts of

$$\sum_{r=0}^{n-1} \exp\left(2\mathrm{i}\left(\theta + \frac{2\pi r}{n}\right)\right)$$

show that $C = \frac{n}{2}$ and $S = 0$. **[7]**