

CORE PURE / COMPLEX NUMBERS

Complex Numbers Basics

Questions by topic

8 questions · 62 marks

Questions

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1. (a) Let $z = x + iy$, where $x, y \in \mathbb{R}$.

(i) Write down z^* , the complex conjugate of z , in terms of x and y . [1]

(ii) Hence show that the product of a complex number and its conjugate is always real. [2]

(b) The complex number w satisfies

$$2w + 12 = 5w^* + 14i$$

(i) Find w , giving your answer in the form $a + bi$, where $a, b \in \mathbb{R}$. [3]

(ii) Calculate $w^2(w^*)^2$. [1]

2. The complex number z satisfies

$$4(z + 2i) = (1 - 2i)z^*$$

where z^* denotes the complex conjugate of z .

Determine z , giving your answer in the form $a + bi$, where a and b are real.

[5]

3. The complex numbers z_1 and z_2 are given by

$$z_1 = \sqrt{3} \left(\cos \left(\frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{6} \right) \right)$$

and

$$z_2 = 2 \left(\cos \left(\frac{5\pi}{6} \right) - i \sin \left(\frac{5\pi}{6} \right) \right)$$

(a) State the exact value of

- (i) $|z_1 z_2|$,
- (ii) the principal argument of $z_1 z_2$, taking $-\pi < \arg(z_1 z_2) \leq \pi$.

[2]

Let $w = z_1 z_2$.

(b) Determine

- (i) the smallest positive integer n for which w^n is a positive real number,
- (ii) the exact value of $|w^n|$ for this value of n .

[3]

4. In this question the principal argument of a complex number lies in the interval $[0, 2\pi)$. Complex numbers z_1 and z_2 are defined by $z_1 = 4 + 3i$ and $z_2 = -12 + 5i$.

- (a) Determine $z_1 z_2$, giving your answer in the form $a + bi$. [2]
- (b) Express z_2 in modulus-argument form. [2]
- (c) Verify, by direct calculation, that $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$. [3]

5. The complex numbers z_1 and z_2 satisfy

$$\begin{aligned}z_1 &= -3 + 2i \\|z_1 z_2| &= 26\sqrt{2} \\ \arg(z_1 z_2) &= -\frac{\pi}{4}\end{aligned}$$

- (a) Express z_1 in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$.
Give r exactly and θ in radians to 4 significant figures.

[2]

- (b) Determine z_2 , giving your answer in the form $a + ib$, where a and b are integers.

[6]

6. The complex number w is given by

$$w = -6\sqrt{3} - 6i$$

(a) (i) Find $|w|$. [2]

(ii) Find the principal argument of w , taking $-\pi < \arg(w) \leq \pi$. [2]

The complex numbers z_1 and z_2 are given by

$$z_1 = a - 3i, \quad z_2 = 2(\cos \theta + i \sin \theta)$$

where a is a positive real constant and $-\pi < \theta \leq \pi$.

(b) Given that $z_1 z_2 = w$,

(i) find the exact value of a . [3]

(ii) find θ , giving your answer as an exact multiple of π . [3]

7. The complex numbers z and w are given by $z = 3 + i$ and $w = 2 + 4i$.

- (a) (i) Find the modulus and principal argument of zw .
(ii) Express zw in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$.

[4]

- (b) The complex number v satisfies

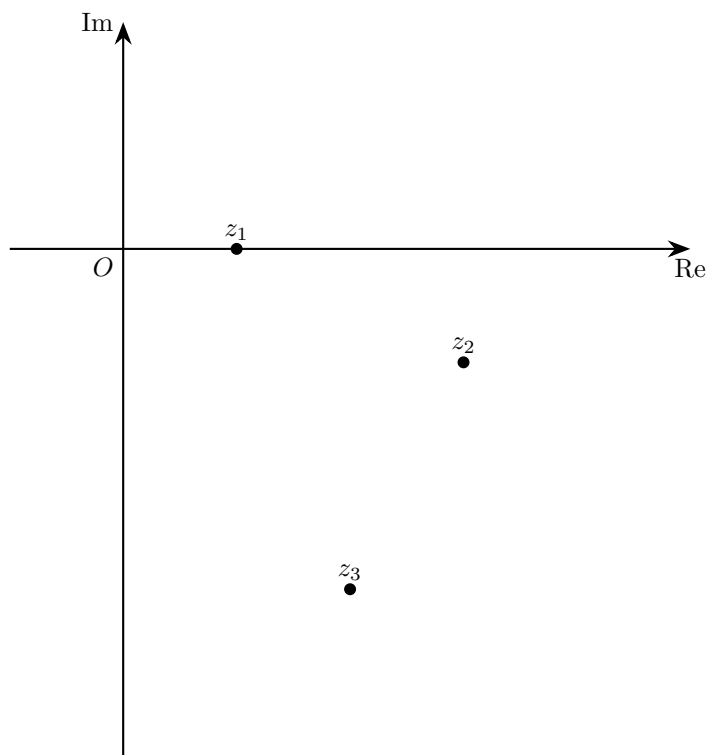
$$\frac{1}{v} = \frac{1}{w} - \frac{1}{z}$$

Find v in the form $a + ib$, where a and b are real.

[5]

- (c) The complex conjugate of v is denoted by v^* .
Show that $vv^* = k$, where k is a real number whose value is to be determined.

[2]



8. The Argand diagram above shows the complex numbers $z_1 = 1$, $z_2 = 3 - i$ and $z_3 = 2 - 3i$.

(a) Explain why no polynomial equation of degree four with real coefficients can have all three of these numbers as roots. [2]

(b) Show that

$$\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \frac{\pi}{4} \quad [3]$$

(c) Hence show that

$$\arctan(3) - \arctan\left(\frac{1}{2}\right) = \frac{\pi}{4} \quad [2]$$

(d) Shade, on a copy of the diagram, the region consisting of all points whose complex coordinates satisfy

$$|z - 1| \leq |z - 2 + 3i| \quad [2]$$