CORE PURE / COMPLEX NUMBERS

Complex Series

Mark scheme 10 questions · 87 marks

EXAM BOARDS Edexcel 9FM0 · AQA 7367 · OCR A H245 · OCR MEI H645 · CIE 9231

Questions

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	Total	**87**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [***n***] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	Geometric series with common ratio z $\lvert z\rvert = \frac{1}{2} < 1$, so the series converges	**E1**	States that the series is geometric, or names z as the common ratio, and that $\lvert z\rvert = \frac{1}{2} < 1$. Not for "$z < 1$" or "the terms get smaller" alone.
		[1]	
1(b)	$z = -\frac{1}{4} + \frac{\sqrt{3}}{4}i$ $\dfrac{1}{1-z} = \dfrac{1}{\frac{5}{4} - \frac{\sqrt{3}}{4}i}$	**M1**	Uses $\frac{1}{1-z}$ with their z substituted, in Cartesian or exponential form. Condone slips in the values of $\cos\frac{2\pi}{3}$ and $\sin\frac{2\pi}{3}$. Not for quoting the formula alone.
	$\dfrac{4}{5-\sqrt{3}\,i} = \dfrac{4\left(5+\sqrt{3}\,i\right)}{\left(5-\sqrt{3}\,i\right)\left(5+\sqrt{3}\,i\right)}$	**dM1**	Multiplies numerator and denominator by the conjugate of their denominator of the form $p+qi$, $q \neq 0$. Condone slips in expanding.
	$= \dfrac{4\left(5+\sqrt{3}\,i\right)}{28} = \dfrac{5+\sqrt{3}\,i}{7}$	**A1 AG**	Correct $\frac{1}{1-z}$ with $z = -\frac{1}{4} + \frac{\sqrt{3}}{4}i$ oe, leading without error to the printed result, with the real denominator 28 or $25+3$ oe seen.
		[3]	
1(c)	$z^n = \dfrac{1}{2^n}\left(\cos\dfrac{2n\pi}{3} + i\sin\dfrac{2n\pi}{3}\right)$ $\displaystyle\sum_{n=0}^{\infty}\frac{1}{2^n}\cos\frac{2n\pi}{3} = \mathrm{Re}\left(\frac{5+\sqrt{3}\,i}{7}\right)$	**M1**	Uses de Moivre's theorem for z^n, soi, and identifies the required sum as the real part of the sum in (b).
	$\dfrac{5}{7}$	**A1**	$\frac{5}{7}$ only, from the real part of the sum in (b). A decimal alone, or $\frac{5}{7}$ with no link to (b), does not score.
		[2]	
	Total for Question 1	**6**	

Question	Scheme	Marks	Notes
2(a)	$C+iS = 1+\frac{1}{4}e^{i\theta}+\frac{1}{16}e^{2i\theta}+\cdots$	**M1**	Forms $C+iS$ and uses Euler's formula to write at least two of the terms in θ as exponentials. Condone coefficient slips.
	Geometric series: first term 1, ratio $\frac{1}{4}e^{i\theta}$	**A1**	Correct first term and common ratio, soi by correct use in the sum formula.
	$C+iS = \dfrac{1}{1-\frac{1}{4}e^{i\theta}}$	**M1**	Uses $\frac{a}{1-r}$ with their first term and their ratio, a multiple of $e^{i\theta}$. Not for quoting the formula alone.
	$= \dfrac{4}{4-e^{i\theta}}$	**A1 AG**	Correct substituted sum leading without error to the printed result. No justification of convergence is needed.
		[4]	
2(b)	$C+iS = \dfrac{4}{(4-\cos\theta)-i\sin\theta}$ $= \dfrac{4\left[(4-\cos\theta)+i\sin\theta\right]}{\cdots}$	**M1**	Multiplies numerator and denominator by the conjugate of their denominator, in exponential or trigonometric form. Condone sign slips.
	Denominator $= (4-\cos\theta)^2+\sin^2\theta$ $= 17-8\cos\theta$	**A1**	Real denominator $17-8\cos\theta$ from a correct expansion.
	$C+iS = \dfrac{16-4\cos\theta+4i\sin\theta}{\text{“}17-8\cos\theta\text{”}}$ $\Rightarrow C = \ldots$ or $S = \ldots$	**M1**	Expands the numerator and equates real or imaginary parts over their real denominator. Condone sign slips.
	$S = \dfrac{4\sin\theta}{17-8\cos\theta}$	**A1**	oe in θ with a real denominator; imaginary parts equated, soi.
	$C = \dfrac{16-4\cos\theta}{17-8\cos\theta}$	**A1 AG**	Correct expression with real denominator leading without error to the printed result, with real parts equated, in words or soi.
		[5]	
	Total for Question 2	**9**	

Question	Scheme	Marks	Notes
3(a)	$C+iS = e^{2i\theta} - \frac{1}{3}e^{5i\theta} + \frac{1}{9}e^{8i\theta} - \cdots$	**M1**	Uses $\cos k\theta + i\sin k\theta = e^{ik\theta}$ to write at least three terms as exponentials.
	Geometric, first term $e^{2i\theta}$, $r = -\frac{1}{3}e^{3i\theta}$	**A1**	Correct common ratio, oe, soi by a correct sum.
	$C+iS = \dfrac{e^{2i\theta}}{1-\left(\text{“}-\frac{1}{3}e^{3i\theta}\text{”}\right)}$	**M1**	Uses $\frac{a}{1-r}$ with their first term and their ratio of the form $ke^{3i\theta}$.
	$= \dfrac{3e^{2i\theta}}{3+e^{3i\theta}}$	**A1 AG**	Printed result from a correct sum without error. No check of $\lvert r\rvert < 1$ needed.
		[4]	
3(b)	$C+iS = \dfrac{3e^{2i\theta}(3+e^{-3i\theta})}{(3+e^{3i\theta})(3+e^{-3i\theta})}$	**M1**	Multiplies numerator and denominator by the conjugate of the denominator, in exponential or trigonometric form.
	Denominator $= 9+3\left(e^{3i\theta}+e^{-3i\theta}\right)+1$ $= 10+6\cos 3\theta$	**A1**	
	Numerator $= 9e^{2i\theta}+3e^{-i\theta}$ $S = \dfrac{\text{Im}\left(9e^{2i\theta}+3e^{-i\theta}\right)}{10+6\cos 3\theta} = \ldots$	**M1**	Expands the numerator and takes its imaginary part over their real denominator. Condone sign slips.
	$S = \dfrac{9\sin 2\theta - 3\sin\theta}{10+6\cos 3\theta}$	**A1 AG**	Printed result from a correct numerator and denominator without error.
		[4]	
	Total for Question 3	**8**	

Question	Scheme	Marks	Notes
4(a)	$C+iS=\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n e^{i(4n+1)\theta}$	**M1**	Combines the two series term by term, using $\cos x+i\sin x=e^{ix}$, into one series of exponentials.
	Geometric, first term $e^{i\theta}$, ratio $-\frac{1}{2}e^{4i\theta}$	**A1**	Common ratio $-\frac{1}{2}e^{4i\theta}$ oe, stated or used.
	$C+iS=\dfrac{e^{i\theta}}{1+\frac{1}{2}e^{4i\theta}}$	**M1**	Uses $\frac{a}{1-r}$ with their first term and ratio, each of the form $ke^{mi\theta}$.
	$=\dfrac{2e^{i\theta}}{2+e^{4i\theta}}$	**A1 AG**	Correct sum to infinity leading without error to the printed result. $\lvert r\rvert<1$ need not be shown.
		[4]	
4(b)	$C+iS=\dfrac{2e^{i\theta}(2+e^{-4i\theta})}{(2+e^{4i\theta})(2+e^{-4i\theta})}$	**M1**	Multiplies numerator and denominator by $2+e^{-4i\theta}$ oe, such as $2+\cos 4\theta-i\sin 4\theta$.
	$(2+e^{4i\theta})(2+e^{-4i\theta})=5+4\cos 4\theta$	**A1**	Correct real denominator from a seen expansion.
	$C=\text{Re}\left(4e^{i\theta}+2e^{-3i\theta}\right)\div$ "$5+4\cos 4\theta$" $=\ldots$	**M1**	Expands the numerator to two exponential terms and takes the real part of each over their real denominator.
	$C=\dfrac{4\cos\theta+2\cos(-3\theta)}{5+4\cos 4\theta}$ $=\dfrac{4\cos\theta+2\cos 3\theta}{5+4\cos 4\theta}$	**A1 AG**	Correct real parts over $5+4\cos 4\theta$ leading without error to the printed result; $\cos(-3\theta)=\cos 3\theta$ may be implied.
		[4]	
	Total for Question 4	**8**	

Question	Scheme	Marks	Notes
5(a)	$C+iS=\sum\cos^n\theta\left(\cos n\theta+i\sin n\theta\right)$ $=\sum\cos^n\theta\left(\cos\theta+i\sin\theta\right)^n$	**M1**	Forms $C+iS$ and uses de Moivre's theorem to write each term with an nth power, oe such as $\cos^n\theta\, e^{in\theta}$. Condone missing limits.
	$C+iS=\sum\left[\cos\theta\left(\cos\theta+i\sin\theta\right)\right]^n$ $r=\cos\theta\left(\cos\theta+i\sin\theta\right)$	**A1**	Terms shown as powers of one quantity, or a constant ratio of consecutive terms, with the correct common ratio identified, oe such as $\cos\theta\, e^{i\theta}$.
	$\lvert r\rvert=\lvert\cos\theta\rvert\times\lvert\cos\theta+i\sin\theta\rvert=\lvert\cos\theta\rvert$ $0<\theta<\pi\Rightarrow\lvert\cos\theta\rvert<1$, so the series converges	**E1**	States that $\lvert r\rvert=\lvert\cos\theta\rvert$, that this is less than 1 for $0<\theta<\pi$, and that the series converges. Not for "$\lvert r\rvert<1$" with no modulus found.
		[3]	
5(b)	$C+iS=\dfrac{1}{1-\text{“}\cos\theta\left(\cos\theta+i\sin\theta\right)\text{”}}$	**M1**	Uses $\frac{a}{1-r}$ with $a=1$ and their common ratio from (a), a complex expression in θ. Not for quoting the formula alone.
	$C+iS=\dfrac{1}{\sin^2\theta-i\sin\theta\cos\theta}$	**A1**	Correct denominator with $1-\cos^2\theta$ replaced by $\sin^2\theta$, oe such as $\sin\theta\left(\sin\theta-i\cos\theta\right)$. Accept a correct equivalent such as $\frac{2}{1-e^{2i\theta}}$.
	Multiplies by $\sin\theta+i\cos\theta$: $C+iS=\dfrac{\sin\theta+i\cos\theta}{\sin\theta\left(\sin^2\theta+\cos^2\theta\right)}$	**M1**	Multiplies numerator and denominator by the conjugate of their denominator, or by $e^{-i\theta}$, oe, to reach a real or purely imaginary denominator, and equates real or imaginary parts. Condone sign slips.
	$C+iS=1+i\cot\theta\Rightarrow S=\cot\theta$	**A1**	Accept $\frac{\cos\theta}{\sin\theta}$; condone $\frac{1}{\tan\theta}$. Not a form with an uncancelled factor of $\sin\theta$. Imaginary parts equated, soi.
	$C=1$	**A1 AG**	Correct sum to infinity leading without error to real part 1, with $C=1$ stated from the real parts; combined steps accepted.
		[5]	
	Total for Question 5	**8**	

Question	Scheme	Marks	Notes
6	$C+iS=\sum_{n=0}^{\infty} r^n e^{i(\alpha+n\beta)}$ $=e^{i\alpha}\sum_{n=0}^{\infty}\left(re^{i\beta}\right)^n$	**M1**	Writes the terms as $r^n e^{i(\alpha+n\beta)}$ and identifies a geometric series with ratio $re^{i\beta}$, soi by its use.
	$C+iS=\frac{e^{i\alpha}}{1-re^{i\beta}}$	**M1 A1**	**M1** Uses $\frac{a}{1-q}$ with their first term and a complex ratio of the form $re^{\pm i\beta}$. Convergence need not be stated. **A1** oe
	$C+iS=\frac{e^{i\alpha}\left(1-re^{-i\beta}\right)}{(1-re^{i\beta})(1-re^{-i\beta})}$	**M1**	Multiplies numerator and denominator by the conjugate of their denominator, in exponential or trigonometric form.
	$\left(1-re^{i\beta}\right)\left(1-re^{-i\beta}\right)$ $=1-r\left(e^{i\beta}+e^{-i\beta}\right)+r^2$ $=1+r^2-2r\cos\beta$	**A1**	
	Equates real and imaginary parts of the numerator: $e^{i\alpha}-re^{i(\alpha-\beta)}$ $=\cos\alpha-r\cos(\alpha-\beta)$ $+i\left(\sin\alpha-r\sin(\alpha-\beta)\right)$	**dM1**	Writes their numerator as real part plus i times imaginary part in terms of α and β, and equates these to C and S. Condone sign slips.
	$S=\frac{\sin\alpha-r\sin(\alpha-\beta)}{1+r^2-2r\cos\beta}$ $C=\frac{\cos\alpha-r\cos(\alpha-\beta)}{1+r^2-2r\cos\beta}$	**A1** **A1 AG**	**A1** oe **A1 AG** From the real part of a correct $C+iS$ over the real denominator; working back from the printed result earns nothing.
	Total for Question 6	**8**	

Question	Scheme	Marks	Notes
7(i)	$\sec\theta(\cos\theta + i\sin\theta)$ $= \sec\theta\cos\theta + i\sec\theta\sin\theta$	**M1**	Expands the bracket, or factorises $1 + i\tan\theta$, using $\sec\theta = \frac{1}{\cos\theta}$ in at least one term.
	$= 1 + i\tan\theta$	**A1 AG**	Both terms simplified without error to the printed result, from either side.
		[2]	
7(ii)	$C + iS = 1 + \binom{n}{1}(i\tan\theta) + \binom{n}{2}(i\tan\theta)^2$ $+ \binom{n}{3}(i\tan\theta)^3 + \cdots$	**M1**	Combines the series as a binomial expansion in $i\tan\theta$, showing at least the terms in $\tan^2\theta$ and $\tan^3\theta$ with their powers of i.
	$C + iS = (1 + i\tan\theta)^n$	**A1**	
	$C + iS = [\sec\theta(\cos\theta + i\sin\theta)]^n$ $= \sec^n\theta(\cos\theta + i\sin\theta)^n$	**M1**	Substitutes part (i) into their $(1 + i\tan\theta)^n$ and takes the power of each factor.
	$C + iS = \sec^n\theta(\cos n\theta + i\sin n\theta)$	**dM1 A1**	**dM1** Applies de Moivre's theorem to $(\cos\theta + i\sin\theta)^n$.
	$S = \sec^n\theta\sin n\theta$ $C = \sec^n\theta\cos n\theta$	**A1** **A1 AG**	**A1** $S = \sec^n\theta\sin n\theta$ stated; the equating of imaginary parts may be implied. **A1 AG** Real parts equated from a correct $C + iS$, with no errors in the part.
		[7]	
	Total for Question 7	**9**	

Question	Scheme	Marks	Notes
8	$\sum_{r=0}^{n-1} \exp\left(2i\theta + \frac{4\pi ir}{n}\right)$ $= \exp(2i\theta) \sum_{r=0}^{n-1} \left(\exp\left(\frac{4\pi i}{n}\right)\right)^r$	**M1**	Writes the sum as a constant times a sum of powers of one complex exponential. Condone slips in the ratio.
	Ratio $\exp\left(\frac{4\pi i}{n}\right) \neq 1$ since $0 < \frac{2}{n} < 1$	**A1**	Correct ratio, with a reason it is not 1 that uses $n > 2$.
	$\exp(2i\theta) \frac{1-\left(\exp\left(\frac{4\pi i}{n}\right)\right)^n}{1-\exp\left(\frac{4\pi i}{n}\right)}$	**M1**	Applies the geometric series sum formula with their first term and ratio. Condone a slip in the number of terms.
	$\left(\exp\left(\frac{4\pi i}{n}\right)\right)^n = \exp(4\pi i) = 1 \Rightarrow \text{sum} = 0$ $\Rightarrow \sum_{r=0}^{n-1} \cos\left(2\theta + \frac{4\pi r}{n}\right) = 0$ $\sum_{r=0}^{n-1} \sin\left(2\theta + \frac{4\pi r}{n}\right) = 0$	**A1**	Shows the ratio to the power n is 1, so the sum is 0, and that both its real and imaginary parts are 0.
	$C = \sum_{r=0}^{n-1} \frac{1+\cos\left(2\theta+\frac{4\pi r}{n}\right)}{2}$ $S = \frac{1}{2}\sum_{r=0}^{n-1} \sin\left(2\theta + \frac{4\pi r}{n}\right)$	**M1**	Uses a double-angle identity to write C or S in terms of the real or imaginary part of the sum. Condone coefficient slips.
	$C = \frac{1}{2}(n) + \frac{1}{2}(0) = \frac{n}{2}$	**A1**	cso, with $\sum_{r=0}^{n-1} 1 = n$ seen or clearly used.
	$S = \frac{1}{2}(0) = 0$	**A1 AG**	Correct identity applied to the zero imaginary part, reaching $S = 0$ with no errors seen.
	Total for Question 8	**7**	

Question	Scheme	Marks	Notes
9(i)	$(1-2e^{2i\theta})(1-2e^{-2i\theta})$ $= 1-2e^{2i\theta}-2e^{-2i\theta}+4$	**M1**	Multiplies out to four terms, using $e^{2i\theta}e^{-2i\theta}=1$. Condone one sign slip.
	$= 5-2(e^{2i\theta}+e^{-2i\theta})$	**A1**	oe, such as $5-2e^{2i\theta}-2e^{-2i\theta}$.
	$= 5-2(2\cos 2\theta) = 5-4\cos 2\theta$	**A1 AG**	Uses $e^{2i\theta}+e^{-2i\theta}=2\cos 2\theta$, or the cosine and sine forms, reaching the printed result without error.
		[3]	
9(ii)	$C+iS = e^{i\theta}+2e^{3i\theta}+\cdots+2^{n-1}e^{(2n-1)i\theta}$	**M1**	Forms $C+iS$ and writes its terms as exponentials; the first two terms and the last are enough.
	Geometric: first term $e^{i\theta}$, ratio $2e^{2i\theta}$	**A1**	Ratio $2e^{2i\theta}$, soi; first term 1 accepted with $e^{i\theta}$ factored out.
	$C+iS = \frac{e^{i\theta}\left(1-(2e^{2i\theta})^n\right)}{1-2e^{2i\theta}}$	**M1 A1**	**M1** Uses $\frac{a(1-r^n)}{1-r}$ or $\frac{a(r^n-1)}{r-1}$ with their first term and ratio, of the form $ke^{2i\theta}$, and n terms. **A1** oe, such as numerator $e^{i\theta}-2^n e^{(2n+1)i\theta}$.
	$= \frac{e^{i\theta}(1-2^n e^{2ni\theta})(1-2e^{-2i\theta})}{(1-2e^{2i\theta})(1-2e^{-2i\theta})}$	**M1**	Multiplies numerator and denominator by the complex conjugate of their denominator.
	$= \frac{e^{i\theta}(1-2^n e^{2ni\theta})(1-2e^{-2i\theta})}{5-4\cos 2\theta}$	**A1**	Denominator $5-4\cos 2\theta$, from part (i); the numerator need not be expanded.
	Numerator $= e^{i\theta}-2e^{-i\theta}-2^n e^{(2n+1)i\theta}$ $+2^{n+1}e^{(2n-1)i\theta}$ Equates real and imaginary parts	**dM1**	Expands their numerator to four terms, writes them in cosine and sine form and takes real and imaginary parts. Condone sign slips.
	$S = \frac{2^{n+1}\sin((2n-1)\theta)-2^n\sin((2n+1)\theta)+3\sin\theta}{5-4\cos 2\theta}$	**A1**	oe.
	$C = \frac{2^{n+1}\cos((2n-1)\theta)-2^n\cos((2n+1)\theta)-\cos\theta}{5-4\cos 2\theta}$	**A1 AG**	Real part of a correct expanded numerator, reaching the printed result without error.
		[9]	
	Total for Question 9	**12**	

Question	Scheme	Marks	Notes
10(i)	$1 - e^{2i\theta} = 1 - \cos 2\theta - i\sin 2\theta$	**M1**	Writes $e^{2i\theta}$ in Cartesian form, as $\cos 2\theta + i\sin 2\theta$ or by expanding $(\cos\theta + i\sin\theta)^2$.
	$= 2\sin^2\theta - 2i\sin\theta\cos\theta$	**M1**	Uses $1 - \cos 2\theta = 2\sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$, or equivalent identities, on both parts. Condone sign slips.
	$= 2\sin\theta(\sin\theta - i\cos\theta)$	**A1 AG**	Printed result reached without error from correct identities, including the sign of the imaginary part.
		[3]	
10(ii)	$C + iS = 1 - \binom{n}{1}e^{2i\theta} + \binom{n}{2}e^{4i\theta} - \cdots$ $= \sum_{r=0}^{n}\binom{n}{r}\left(-e^{2i\theta}\right)^r$	**M1**	Combines the series into terms $\binom{n}{r}(-1)^r e^{2ri\theta}$, at least three seen or in general, and identifies a binomial expansion. Condone sign slips.
	$C + iS = \left(1 - e^{2i\theta}\right)^n$	**A1**	
	$C + iS = [2\sin\theta(\sin\theta - i\cos\theta)]^n$ $= 2^n\sin^n\theta\,(\sin\theta - i\cos\theta)^n$	**M1**	Substitutes the result of (i) into $\left(1 - e^{2i\theta}\right)^n$ and separates the real factor $2^n\sin^n\theta$.
	$\sin\theta - i\cos\theta = -i(\cos\theta + i\sin\theta)$	**M1**	Writes $\sin\theta - i\cos\theta$ as a multiple of $\cos\theta + i\sin\theta$, or in exponential or modulus-argument form.
	$\sin\theta - i\cos\theta = e^{-i\pi/2}(\cos\theta + i\sin\theta)$	**A1**	oe, such as $e^{i(\theta-\pi/2)}$ or $\cos\left(\theta - \frac{\pi}{2}\right) + i\sin\left(\theta - \frac{\pi}{2}\right)$.
	$(\sin\theta - i\cos\theta)^n = e^{-in\pi/2}\,e^{in\theta}$	**dM1**	Applies de Moivre's theorem, or the laws of indices, to raise their exponential or modulus-argument form to the power n.
	$C + iS = 2^n\sin^n\theta\,e^{i(n\theta - n\pi/2)}$ $= 2^n\sin^n\theta\left(\cos\left(n\theta - \frac{n\pi}{2}\right)\right.$ $\left. + i\sin\left(n\theta - \frac{n\pi}{2}\right)\right)$	**A1**	In exponential or modulus-argument form.
	$S = 2^n\sin^n\theta\sin\left(n\theta - \frac{n\pi}{2}\right)$ $C = 2^n\sin^n\theta\cos\left(n\theta - \frac{n\pi}{2}\right)$	**A1** **A1 AG**	**A1** oe. **A1 AG** Equates real parts of a correct expression for $C + iS$, reaching the printed form.
		[9]	
	Total for Question 10	**12**	