CORE PURE / COMPLEX NUMBERS

# Complex Numbers Basics

Mark scheme 17 questions · 120 marks

EXAM BOARDS Edexcel 9FM0 · AQA 7367 · OCR A H245 · OCR MEI H645 · CIE 9231

## Questions

| Question | Marks by part | Marks | Page |
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| Question 1 | (a)(i) 1 (a)(ii) 2 (a)(iii) 3 (b) 3 | 9 | 4 |
| Question 2 | (a) 3 (b) 3 | 6 | 5 |
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| Question 17 | (a) 2 (b) 3 (c) 2 (d) 2 | 9 | 20 |
| | **Total** | **120** | |

# Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

### Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

| Code | Meaning |
|---|---|
| **M1** | Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted. |
| **dM1** | Dependent method: as M1, but only if the method mark before it was earned, unless its note names another. |
| **A1** | Accuracy: a correct answer or step. It needs the method mark it follows. |
| **A1ft** | As A1, following through from an earlier answer that the note names. |
| **A1 AG** | The last mark of a part whose answer is printed in the question: complete working, with no errors. |
| **B1** | A correct value, fact or statement. It needs no method mark. |
| **B1ft** | As B1, following through from an earlier answer that the note names. |
| **E1** | A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark. |

### Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

| | | | |
|---|---|---|---|
| **ft** | follow through<br>Correct work from the student's own earlier wrong value earns the mark. | **AG** | answer given<br>The answer is printed in the question, so every step to it must be shown. |
| **cao** | correct answer only<br>No follow-through from an earlier error. | **cso** | correct solution only<br>No error anywhere in the part, not just in its answer. |
| **oe** | or equivalent | **awrt** | answers which round to |
| **soi** | seen or implied<br>Earned if written, or if later working shows it was used. | **isw** | ignore subsequent working<br>A correct answer, once reached, isn't spoiled by later working. |
| **SC** | special case<br>Credit, as the note says, for work outside the main method. | **their** | the student's own earlier value |
| **condone** | a slip that doesn't cost the mark | [ ] | working the mark doesn't need |
| **s.f.** | significant figures | **d.p.** | decimal places |

### Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

### Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

### Value of $g$

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state $g$, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **1(a)(i)** | $2z - w = 7 - 8i$ | **B1** | |
| | | **[1]** | |
| **1(a)(ii)** | $(4 - 3i)(1 + 2i) = 4 + 8i - 3i - 6i^2$ | **M1** | Expands to four terms and uses $i^2 = -1$. Condone one slip. |
| | $= 10 + 5i$ | **A1** | |
| | | **[2]** | |
| **1(a)(iii)** | $\frac{z}{w} = \frac{(4 - 3i)(1 - 2i)}{(1 + 2i)(1 - 2i)}$ | **M1** | Multiplies numerator and denominator by $1 - 2i$, or a multiple of it, and attempts to expand. |
| | $= \frac{-2 - 11i}{5}$ | **A1** | Correct numerator $-2 - 11i$, or the matching multiple of it for a multiple of $1 - 2i$. |
| | $= -\frac{2}{5} - \frac{11}{5}i$ | **A1** | oe such as $-0.4 - 2.2i$. $\frac{-2-11i}{5}$ alone scores A0. |
| | | **[3]** | |
| **1(b)** | $p(4 - 3i) + q(1 + 2i) = 10 - 13i$<br>Equates real and imaginary parts:<br>$4p + q = 10, \quad -3p + 2q = -13$ | **M1** | Substitutes $z$ and $w$ and equates real and imaginary parts to form two equations in $p$ and $q$. Condone sign slips. |
| | $\Rightarrow p = \ldots, \; q = \ldots$ | **dM1** | Solves their two linear equations for both $p$ and $q$, by any method, including a calculator. |
| | $p = 3, \quad q = -2$ | **A1** | |
| | | **[3]** | |
| | **Total for Question 1** | **9** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **2(a)** | $(z-3)^2 - 9 + 34 = 0$<br>$\Rightarrow (z-3)^2 = \ldots \Rightarrow z = 3 \pm \sqrt{\ldots}$ | **M1** | Completes the square, or uses the correct formula with their values, leading to $z = \ldots$. Condone sign slips. |
| | $(z-3)^2 = -25$ | **A1** | Or $\sqrt{-100} = 10i$ oe in the formula. |
| | $z = 3 + 5i, \quad z = 3 - 5i$ | **A1** | Both roots, in the form $a + bi$; $3 \pm 5i$ accepted. |
| | | **[3]** | |
| **2(b)** | $(z^2 + 9)(z^2 - 4) = 0 \Rightarrow z^2 = \ldots$ | **M1** | Solves a three-term quadratic in $z^2$, or in a substituted letter, by any method, leading to $z^2 = \ldots$. |
| | $z^2 = 4$ or $z^2 = -9$ | **A1** | Either correct value of $z^2$. |
| | $z = \pm 2, \quad z = \pm 3i$ | **A1** | All four roots and no others. |
| | | **[3]** | |
| | **Total for Question 2** | **6** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **3(a)(i)** | $z^* = x - iy$ | **B1** | |
| | | **[1]** | |
| **3(a)(ii)** | $zz^* = (x+iy)(x-iy)$<br>$= x^2 - i^2y^2$ | **M1** | Expands $(x+iy)(x-iy)$ and uses $i^2 = -1$. Cross terms may cancel unseen; condone one sign slip. |
| | $= x^2 + y^2$, which is real | **A1 AG** | Correct expansion leading without error to $x^2 + y^2$, with a statement that it is real. |
| | | **[2]** | |
| **3(b)(i)** | $w = x + iy, \quad w^* = x - iy$, where $x, y \in \mathbb{R}$<br>$2x + 12 + 2iy = 5x + (14 - 5y)i$ | **M1** | Substitutes $w = x + iy$ and $w^* = x - iy$ and equates real and imaginary parts. Alternatively, conjugates the whole equation to obtain $2w^* + 12 = 5w - 14i$, including the sign change of $14i$. Condone one sign slip. |
| | $2x + 12 = 5x, \quad 2y = 14 - 5y$<br>$\Rightarrow x = 4, \quad y = 2$ | **dM1** | Solves their real and imaginary equations for $x$ and $y$. For the conjugation method, solves their two equations simultaneously, eliminating $w^*$, to reach $w = \ldots$. Condone arithmetic slips. |
| | $w = 4 + 2i$ | **A1** | In the form $a + bi$ only; $a = 4$, $b = 2$ alone does not score. |
| | | **[3]** | |
| **3(b)(ii)** | $w^2(w^*)^2 = (12 + 16i)(12 - 16i) = 400$ | **B1** | |
| | | **[1]** | |
| | **Total for Question 3** | **7** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **4(a)** | $f(3) = 3^3 - 7 \times 3^2 + 17 \times 3 - 15$ | **M1** | Substitutes $z = 3$ into $f(z)$. Division by $(z-3)$ attempted to a remainder is also accepted. |
| | $= 27 - 63 + 51 - 15 = 0$<br>$f(3) = 0$, so $(z-3)$ is a factor | **A1 AG** | Correct substitution evaluated to 0, or zero remainder from correct division, with a conclusion that $(z-3)$ is a factor. A0 if the conclusion is missing. |
| | | **[2]** | |
| **4(b)** | $z^3 - 7z^2 + 17z - 15 = (z-3)\left(z^2 + pz + q\right)$<br>$\Rightarrow p = \ldots,\ q = \ldots$ | **M1** | Divides $f(z)$ by $(z-3)$, compares coefficients or writes the factor by inspection, to reach a three-term quadratic factor. Condone slips. |
| | $f(z) = (z-3)\left(z^2 - 4z + 5\right)$ | **A1** | |
| | "$z^2 - 4z + 5$" $= 0$<br>$\Rightarrow z = \dfrac{4 \pm \sqrt{16 - 20}}{2} = \ldots$ | **M1** | Solves their three-term quadratic factor by formula, completing the square or calculator, leading to $z = \ldots$. Not for solving the cubic directly by calculator. |
| | $z = 3, \quad z = 2 + i, \quad z = 2 - i$ | **A1** | All three roots, the complex ones in the form $a + bi$; $2 \pm i$ accepted. |
| | | **[4]** | |
| **Total for Question 4** | | **6** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **5** | $4(a+bi+2i)=(1-2i)(a-bi)$<br>$\Rightarrow 4a+(4b+8)i=(a-2b)+(-2a-b)i$ | **M1** | Substitutes $z=a+bi$ and $z^*=a-bi$ and expands the product using $i^2=-1$. Condone sign slips. |
| | $4a=a-2b$<br>$4b+8=-2a-b$ | **M1 A1** | **M1** Equates real and imaginary parts of their expansion to form two equations in $a$ and $b$.<br>**A1** Correct pair, oe. |
| | $3a+2b=0,\ 2a+5b=-8\Rightarrow a=\ldots,\ b=\ldots$ | **dM1** | Solves their two linear equations for $a$ and $b$ by any method; a calculator may be used. |
| | $z=\frac{16}{11}-\frac{24}{11}i$ | **A1** | oe, or awrt $1.45-2.18i$. Must be given as $z$ in the form $a+bi$; $a$ and $b$ stated separately do not score. |
| | **Total for Question 5** | **5** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **6** | $(a+bi)^2 = \left(a^2 - b^2\right) + 2abi$<br>$a^2 - b^2 = 21, \quad 2ab = -20$ | **M1 A1** | **M1** Squares $a+bi$ and equates real and imaginary parts with $21-20i$. Condone sign slips.<br>**A1** Both equations, oe. |
| | $a^2 - \frac{100}{a^2} = 21$<br>$\Rightarrow a^4 - 21a^2 - 100 = 0 \Rightarrow a^2 = \ldots$ | **dM1** | Eliminates one unknown to a three-term quadratic in $a^2$ or $b^2$ and solves it by any means, or uses $a^2 + b^2 = \lvert 21 - 20i \rvert$. Condone slips. |
| | $a^2 = 25 \ \left[a^2 \neq -4\right] \Rightarrow a = \pm 5$ | **A1** | $a = \pm 5$, or $b = \pm 2$, with the negative value of $a^2$ or $b^2$ rejected or not used. |
| | $z = 5 - 2i, \quad z = -5 + 2i$ | **A1** | Both, and no others; $\pm(5-2i)$ accepted. |
| | **Total for Question 6** | **5** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **7(a)** | $2+3i$ | **B1** | |
| | | **[1]** | |
| **7(b)** | $(z-(2-3i))(z-(2+3i))$<br>$=(z-2)^2-(3i)^2$ | **M1** | Multiplies out the factors from the conjugate pair, or uses their sum and product, to form a quadratic. Condone sign slips. |
| | $=z^2-4z+13$ | **A1** | |
| | $z^3+pz^2+qz+13$<br>$=(\text{“}z^2-4z+13\text{”})(z+b)$<br>$\Rightarrow 13b=13 \Rightarrow b=\ldots$ | **M1** | Writes the cubic as their quadratic times a linear factor and compares a coefficient, or uses a root relationship such as product of roots $=-13$. |
| | Third root $z=-1$ | **A1** | |
| | $(z^2-4z+13)(z+1)=z^3-3z^2+9z+13$<br>$p=-3, \quad q=9$ | **A1** | Both values. |
| | | **[5]** | |
| | **Total for Question 7** | **6** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **8(a)(i)** | $\lvert z_1\rvert = 2$ | **B1** | |
| | $\arg z_1 = \frac{\pi}{3}$ | **B1** | |
| | | **[2]** | |
| **8(a)(ii)** | $\lvert z_2\rvert = 6$ | **B1** | |
| | $\arg z_2 = \frac{5\pi}{6}$ | **B1** | |
| | | **[2]** | |
| **8(b)(i)** | $r = \sqrt{(-5)^2 + (-5)^2}$ $[= 5\sqrt{2}]$<br>$\theta = -\left(\pi - \arctan\frac{5}{5}\right) = \ldots$ | **M1** | Attempts the modulus by Pythagoras and an argument placed in the third quadrant, by any method. Condone $\frac{5\pi}{4}$, an unsimplified modulus or a slip in the modulus. |
| | $z_3 = 5\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$ | **A1** | Values written into the form, $\theta$ in range. $r = \sqrt{50}$ accepted. |
| | | **[2]** | |
| **8(b)(ii)** | $r = \sqrt{\left(4\sqrt{3}\right)^2 + (-4)^2}$ $[= 8]$<br>$\theta = -\arctan\frac{4}{4\sqrt{3}} = \ldots$ | **M1** | Attempts the modulus by Pythagoras and an argument placed in the fourth quadrant, by any method. Condone $\frac{11\pi}{6}$, an unsimplified modulus or a slip in the modulus. |
| | $z_4 = 8\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$ | **A1** | Values written into the form, $\theta$ in range. |
| | | **[2]** | |
| | **Total for Question 8** | **8** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **9(a)** | $z_1 = 6\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ | **M1** | Evaluates both trigonometric values, exact or decimal, and multiplies by 6. Condone one wrong value or sign. Implied by a correct exact answer. |
| | $z_1 = 3 + 3\sqrt{3}\,i$ | **A1** | oe exact, such as $3 + \sqrt{27}\,i$. |
| | | **[2]** | |
| **9(b)** | $z_2 = 4\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$ | **M1** | Evaluates both trigonometric values, exact or decimal, and multiplies by 4. Condone one wrong value or sign. Implied by a correct exact answer. |
| | $z_2 = -2\sqrt{2} + 2\sqrt{2}\,i$ | **A1** | oe exact, such as $-\sqrt{8} + \sqrt{8}\,i$. |
| | | **[2]** | |
| **9(c)** | $z_3 = 2\sqrt{3}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$ | **M1** | Evaluates both trigonometric values, exact or decimal, and multiplies by $2\sqrt{3}$. Condone one wrong value or sign. Implied by a correct exact answer. |
| | $z_3 = -3 - \sqrt{3}\,i$ | **A1** | cso; both signs negative. |
| | | **[2]** | |
| | **Total for Question 9** | **6** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **10(a)** | $\lvert zw\rvert = 8 \times 2 = 16$ | **B1** | |
| | $\arg(zw) = \frac{11\pi}{12} + \frac{\pi}{4} = \frac{7\pi}{6}$ | **M1** | Adds the two arguments. Condone arithmetic slips; the adjustment into range is not needed for this mark. |
| | $zw = 16\left(\cos\left(-\frac{5\pi}{6}\right) + i\sin\left(-\frac{5\pi}{6}\right)\right)$ | **A1** | Values written into the given form, with $\theta = -\frac{5\pi}{6}$ or awrt $-2.62$. $\theta = \frac{7\pi}{6}$ scores A0. |
| | | **[3]** | |
| **10(b)** | $\left\lvert\frac{z}{w}\right\rvert = \frac{8}{2}, \quad \arg\left(\frac{z}{w}\right) = \frac{11\pi}{12} - \frac{\pi}{4}$ | **M1** | Divides the moduli and subtracts the argument of $w$ from that of $z$. Condone arithmetic slips. |
| | $\frac{z}{w} = 4\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$ | **A1** | Modulus 4 and argument $\frac{2\pi}{3}$, soi. |
| | $= -2 + 2\sqrt{3}\,i$ | **A1** | Or $a = -2$, $b = 2\sqrt{3}$ stated. |
| | | **[3]** | |
| | **Total for Question 10** | **6** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **11(a)** | $\lvert z^2\rvert = \left(\sqrt{2}\right)^2 = \ldots$, $\arg(z^2) = 2 \times \frac{5\pi}{12} = \ldots$ | **M1** | Squares the modulus and doubles the argument, or expands $z \times z$ by any valid method. Condone one slip. |
| | $z^2 = 2\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)$ | **A1** | Modulus 2 and argument $\frac{5\pi}{6}$, implied by a correct final answer. |
| | $z^2 = -\sqrt{3} + i$ | **A1** | Or $a = -\sqrt{3}$, $b = 1$ stated. |
| | | **[3]** | |
| **11(b)** | $\lvert z^3\rvert = 2\sqrt{2}$ | **B1** | Or $\sqrt{8}$ oe, or awrt 2.83. |
| | $\arg(z^3) = \frac{5\pi}{6} + \frac{5\pi}{12} = \frac{5\pi}{4}$ | **M1** | Adds $\frac{5\pi}{12}$ to their $\arg(z^2)$, or triples $\frac{5\pi}{12}$. Condone arithmetic slips; the adjustment into range is not needed for this mark. |
| | $\arg(z^3) = -\frac{3\pi}{4}$ | **A1** | Or awrt $-2.36$. |
| | | **[3]** | |
| **Total for Question 11** | | **6** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **12(a)** | (i) $\lvert z_1 z_2 \rvert = 2\sqrt{3}$ | **B1** | |
| | (ii) $\arg(z_1 z_2) = -\frac{2\pi}{3}$ | **B1** | Must lie in $-\pi < \theta \leq \pi$. |
| | | **[2]** | |
| **12(b)** | $\arg(w^n) = n\left(\text{“}-\frac{2\pi}{3}\text{”}\right) = 2k\pi \Rightarrow n = \ldots$ | **M1** | Multiplies their $\arg w$ by $n$ and sets it equal to a multiple of $2\pi$, or finds arguments of successive powers until one is. Not for a multiple of $\pi$ alone. |
| | (i) $n = 3$ | **A1** | |
| | (ii) $\lvert w^3 \rvert = (\text{“}2\sqrt{3}\text{”})^3 = 24\sqrt{3}$ | **B1ft** | ft their $\lvert z_1 z_2 \rvert$ raised to their $n$. |
| | | **[3]** | |
| | **Total for Question 12** | **5** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **13(a)** | $(4+3i)(-12+5i) = -48+20i-36i+15i^2$ | **M1** | Expands to four terms and uses $i^2=-1$. Condone one sign slip. |
| | $-63-16i$ | **A1** | |
| | | **[2]** | |
| **13(b)** | $\lvert z_2\rvert = \sqrt{(-12)^2+5^2} = 13$<br>$\arg z_2 = \pi - \arctan\frac{5}{12}$ | **M1** | Attempts the modulus and an argument placed in the second quadrant, such as $\pi - \arctan\frac{5}{12}$. Condone a slip in the modulus. |
| | $z_2 = 13(\cos\theta + i\sin\theta)$<br>$\theta = \pi - \arctan\frac{5}{12}$ $[\approx 2.75]$ | **A1** | Form $13(\cos\theta+i\sin\theta)$ with $\theta$ exact oe or awrt 2.75. |
| | | **[2]** | |
| **13(c)** | $\arg z_1 = \arctan\frac{3}{4}$<br>$\arg z_2 = \pi - \arctan\frac{5}{12}$<br>$\arg(z_1z_2) = \pi + \arctan\frac{16}{63}$ | **B1** | All three, exact oe or awrt 0.644, 2.75, 3.39, each in $0 \le \arg z < 2\pi$ and its correct quadrant. |
| | $\tan\left(\arctan\frac{3}{4} - \arctan\frac{5}{12}\right)$<br>$= \dfrac{\frac{3}{4}-\frac{5}{12}}{1+\frac{3}{4}\times\frac{5}{12}}$ | **M1** | Applies the compound angle formula for tan of a sum or difference of their angles, with their tangents substituted. A decimal comparison alone earns neither this nor the A1. |
| | $= \frac{16}{63}$<br>$\arg z_1 + \arg z_2 = \pi + \arctan\frac{16}{63}$<br>$= \arg(z_1z_2)$ | **A1 AG** | Exact $\frac{16}{63}$ obtained without error, then a statement that the sum of arguments equals $\arg(z_1z_2)$. Condone no comment on the range of the angle. |
| | | **[3]** | |
| | **Total for Question 13** | **7** | |

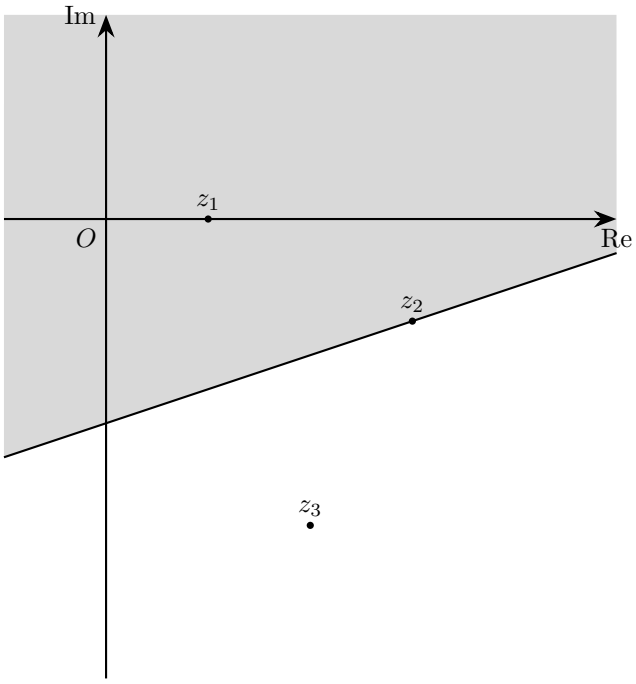
| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **14(a)** | $r = \sqrt{(-3)^2 + 2^2}$<br>$\theta = \pi - \arctan\frac{2}{3}$ | **M1** | Attempts the modulus by Pythagoras and an argument between $\frac{\pi}{2}$ and $\pi$ using $\arctan\frac{2}{3}$ or $\arctan\frac{3}{2}$. |
| | $z_1 = \sqrt{13}\,(\cos 2.554 + i \sin 2.554)$ | **A1** | $r = \sqrt{13}$ exactly and $\theta$ awrt 2.554, in the required form. |
| | | **[2]** | |
| **14(b)** | $z_1 z_2 = (-3 + 2i)(a + bi)$<br>$= (-3a - 2b) + (2a - 3b)i$ | **M1 A1** | **M1** Expands $(-3 + 2i)(a + bi)$ using $i^2 = -1$. Condone one sign slip.<br>**A1** Correct real and imaginary parts, oe. |
| | $2a - 3b = -(-3a - 2b) \Rightarrow a = \ldots$ | **M1** | Uses $\arg = -\frac{\pi}{4}$ to set their imaginary part equal to minus their real part, or their ratio equal to $\tan(-\frac{\pi}{4})$, and finds a relation between $a$ and $b$. |
| | $a = -5b$ | **A1** | |
| | $\sqrt{(13b)^2 + (-13b)^2} = 26\sqrt{2} \Rightarrow b = \ldots$ | **dM1** | Substitutes their relation into a modulus equation, for $z_1 z_2$ with $26\sqrt{2}$ or for $z_2$ with $2\sqrt{26}$, and solves for $b$. |
| | $z_2 = -10 + 2i$ | **A1** | cao. A0 if $10 - 2i$ is also given as an answer. |
| | | **[6]** | |
| | **Total for Question 14** | **8** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **15(a)(i)** | $\lvert w\rvert = \sqrt{(-6\sqrt{3})^2 + (-6)^2}$ | **M1** | Takes the square root of the sum of the squares of the real and imaginary parts. Condone sign slips. |
| | $\lvert w\rvert = 12$ | **A1** | |
| | | **[2]** | |
| **15(a)(ii)** | $\arg w = -\pi + \arctan\left(\frac{6}{6\sqrt{3}}\right)$ | **M1** | Uses arctan of a ratio of the parts' magnitudes and adjusts for the third quadrant, e.g. $-\pi + \alpha$ or $-\frac{\pi}{2} - \beta$. |
| | $\arg w = -\frac{5\pi}{6}$ | **A1** | $-\frac{5\pi}{6}$ or awrt $-2.62$, in the range $-\pi < \arg w \leq \pi$. |
| | | **[2]** | |
| **15(b)(i)** | $z_1 z_2 = 2(a\cos\theta + 3\sin\theta) + 2(a\sin\theta - 3\cos\theta)i$<br>$a\cos\theta + 3\sin\theta = -3\sqrt{3}$<br>$a\sin\theta - 3\cos\theta = -3$ | **M1** | Expands $z_1 z_2$ and equates real and imaginary parts with $w$, giving two equations; condone sign slips. Or uses $\lvert z_1\rvert = \frac{\lvert w\rvert}{\lvert z_2\rvert}$. |
| | $(a\cos\theta + 3\sin\theta)^2 + (a\sin\theta - 3\cos\theta)^2$<br>$= 27 + 9 \Rightarrow a^2 + 9 = 36 \Rightarrow a = \ldots$ | **dM1** | Squares and adds their equations to eliminate $\theta$, or sets $a^2 + 9$ equal to their $\lvert z_1\rvert^2$, leading to $a = \ldots$ |
| | $a = 3\sqrt{3}$ | **A1** | $3\sqrt{3}$ or $\sqrt{27}$. A0 if the negative root is also given as an answer. |
| | | **[3]** | |
| **15(b)(ii)** | $z_2 = \frac{-6\sqrt{3}-6i}{\text{“}3\sqrt{3}\text{”}-3i} \times \frac{\text{“}3\sqrt{3}\text{”}+3i}{\text{“}3\sqrt{3}\text{”}+3i}$ | **M1** | Divides $w$ by $z_1$ with their $a$, multiplying numerator and denominator by the conjugate of the denominator. Condone sign slips. |
| | $z_2 = -1 - \sqrt{3}\,i$ | **A1** | Or $\cos\theta = -\frac{1}{2}$ and $\sin\theta = -\frac{\sqrt{3}}{2}$. |
| | $\theta = -\pi + \arctan\sqrt{3} = -\frac{2\pi}{3}$ | **A1** | In the range $-\pi < \theta \leq \pi$. |
| | | **[3]** | |
| | **Total for Question 15** | **10** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **16(a)** | $zw = (3+i)(2+4i)$<br>$= 6 + 12i + 2i + 4i^2$ | **M1** | Expands the product using $i^2 = -1$. Condone one arithmetic slip. |
| | $zw = 2 + 14i$ | **A1** | |
| | $\lvert zw\rvert = \sqrt{2^2 + 14^2} = \ldots$<br>$\arg(zw) = \arctan\frac{14}{2} = \ldots$ | **M1** | Finds the modulus and argument of their $zw$, the argument in the correct quadrant for their $zw$. |
| | $zw = 10\sqrt{2}\,(\cos(\arctan 7)$<br>$+ i\sin(\arctan 7))$ | **A1** | Accept $\sqrt{200}$ or awrt 14.1, and $\theta$ awrt 1.43. |
| | | **[4]** | |
| **16(b)** | $\frac{1}{v} = \frac{z-w}{zw} = \frac{(3+i)-(2+4i)}{\text{“}2+14i\text{”}}$ | **M1** | Combines over a common denominator, or finds $\frac{1}{w}$ and $\frac{1}{z}$ in the form $a + bi$ and subtracts. |
| | $\frac{1}{v} = \frac{1-3i}{2+14i}$ | **A1** | oe, such as $-\frac{2+i}{10}$. |
| | $v = \frac{(2+14i)(1+3i)}{(1-3i)(1+3i)}$ | **M1** | Inverts their $\frac{1}{v}$ and multiplies numerator and denominator by the conjugate of the denominator. |
| | $= \frac{2+6i+14i+42i^2}{1-9i^2}$ | **dM1** | Expands numerator and denominator using $i^2 = -1$, reaching a real denominator. Condone one slip. |
| | $v = -4 + 2i$ | **A1** | |
| | | **[5]** | |
| **16(c)** | $vv^* = (-4+2i)(-4-2i) = \ldots$ | **M1** | Multiplies their $v$ by its conjugate, or uses $vv^* = a^2 + b^2$ with their $a$ and $b$. |
| | $vv^* = 20$, so $k = 20$ | **A1** | |
| | | **[2]** | |
| | **Total for Question 16** | **11** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **17(a)** | Real coefficients $\Rightarrow 3+i$ and $2+3i$ also roots | **E1** | States that non-real roots occur in conjugate pairs, so the conjugates of $z_2$ and $z_3$ must also be roots. |
| | Five distinct roots, but a quartic has at most four | **E1** | Concludes that five distinct roots, or more than four, is impossible for a degree-four polynomial. |
| | | **[2]** | |
| **17(b)** | $\frac{2-i}{1-3i} \times \frac{1+3i}{1+3i} = \ldots$ | **M1** | Substitutes and multiplies numerator and denominator by the conjugate of their denominator. Condone sign slips in forming the differences. |
| | $= \frac{1}{2} + \frac{1}{2}i$ | **A1** | oe, such as $\frac{5+5i}{10}$. |
| | First quadrant $\Rightarrow \arg = \arctan 1 = \frac{\pi}{4}$ | **A1 AG** | Correct quotient leading without error to $\frac{\pi}{4}$, with the first quadrant, or both parts positive, stated. |
| | | **[3]** | |
| **17(c)** | $\arg\left(\frac{z_2-z_1}{z_3-z_1}\right) = \arg(2-i) - \arg(1-3i)$<br>$= -\arctan\frac{1}{2} - (-\arctan 3)$ | **M1** | Uses the argument of a quotient as the difference of arguments, with arctan forms of both arguments placed in the fourth quadrant. Condone one sign slip. |
| | $\frac{\pi}{4} = \arctan 3 - \arctan\frac{1}{2}$ | **A1 AG** | Correct negative arguments equated to $\frac{\pi}{4}$ from (b), reaching the printed result without error. |
| | | **[2]** | |

*Question 17 continued*

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **17(d)** |  | | |
| | *Shape* | **B1** | Straight line through the midpoint of $z_1$ and $z_3$, visibly perpendicular to their join. Intercepts need not be marked. |
| | *Shading* | **B1** | Half-plane containing $z_1$ shaded, on a line that is at least roughly the bisector. Condone a dashed boundary. |
| | | **[2]** | |
| | **Total for Question 17** | **9** | |