ESAT-style practice paper

Set C Maths 2

Mathematics 2

TIME ALLOWED
40
MINUTES

27 multiple-choice questions

An average of 89 seconds per question.

Instructions

- Attempt all 27 questions.
- Calculators are *not* permitted.
- No formula booklet is provided.
- Use scrap paper or a whiteboard for rough working.

Information

- Each question has one correct answer and is worth one mark.
- No marks are deducted for incorrect answers.
- The questions are not arranged in order of difficulty.
- The ESAT itself is taken on a computer, with erasable sheets for rough working.

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Polynomials

1. Find the smallest positive integer n for which

$$n^2 - 20n + 91 < 0$$

A 7

B 8

C 9

D 10

E 11

F 12

Geometry

2. A circle has centre O and radius 5.

Points A and B lie on the circumference.

The area of triangle AOB is 10.

What is the greatest possible length of chord AB?

A $4\sqrt{5}$

B $2\sqrt{5}$

C $5\sqrt{2}$

D 8

E 6

F $7\sqrt{3}$

Differentiation

3. A curve has equation

$$y = x^2 - 4\sqrt{x}$$

for $x > 0$.

The tangent to the curve at the point where $x = 4$ meets the x-axis at P and the y-axis at Q.

O is the origin.

What is the area of triangle OPQ?

A $\frac{100}{7}$

B $\frac{64}{3}$

C $\frac{200}{7}$

D $\frac{1058}{31}$

E $\frac{400}{7}$

F $\frac{1800}{7}$

Coordinate Geometry

4. Let S be the region of the (x, y)-plane satisfying

$$1 \leq x^2 + y^2 \leq 16 \quad \text{and} \quad y \geq |x|$$

The total area of S is

A $\dfrac{15\pi}{8}$

B $\dfrac{15\pi}{4}$

C $\dfrac{45\pi}{8}$

D $\dfrac{15\pi}{2}$

E $\dfrac{45\pi}{4}$

Exponentials & Logarithms

5. The real numbers x and y satisfy both of the following equations:

$$\frac{9^x}{3^y} = 27$$

$$\log_3 x + \log_3(y+6) = 2$$

What is the value of $x + y$?

A -12

B $-\frac{21}{2}$

C $-\frac{15}{2}$

D 0

E $\frac{3}{2}$

F 6

Sequences & Series

6. An arithmetic progression $a_1, a_2, a_3, \ldots$ and a geometric progression $g_1, g_2, g_3, \ldots$ each have real terms.

The two progressions satisfy

$$a_1 = g_1, \quad a_2 = g_2, \quad \text{and} \quad a_4 = g_4$$

Neither progression is constant.

What is the value of $\dfrac{a_3}{g_3}$?

A $-\dfrac{5}{4}$

B $-\dfrac{4}{5}$

C $-\dfrac{3}{4}$

D $\dfrac{3}{4}$

E $\dfrac{4}{5}$

F $\dfrac{5}{4}$

Inequalities

7. The inequality

$$(k-2)x^2 + 4x + k + 1 < 0$$

holds for every real value of x, where k is a real constant.

What is the complete set of possible values of k?

A $k < -2$

B $k < 2$

C $k > 3$

D $-2 < k < 2$

E $-2 < k < 3$

F $k < -2$ or $k > 3$

Integration

8. For each positive integer n, define

$$F(n) = \int_0^1 x^n \, \mathrm{d}x$$

and

$$H(n) = \sum_{r=1}^{n} \frac{1}{F(r)}$$

What is the smallest positive integer n such that $H(n) > 15$?

A 3

B 4

C 5

D 6

E 7

Polynomials

9. The equation

$$x^3 - 3x + p = 0$$

where p is a real constant, has three distinct real solutions for x.

Find the complete set of values of p

A $-3 < p < 3$

B $-2 < p < 2$

C $-2 \leq p \leq 2$

D $p < -2$ or $p > 2$

E $0 < p < 4$

F $-4 < p < 0$

Integration

10. For a real number a, let

$$I(a) = \int_0^3 x(x-2)(x-a)\,\mathrm{d}x$$

What is the value of $I(a)$?

A $-\frac{9}{4}$

B 0

C $\frac{9}{4}$

D $\frac{9}{2}$

E $\frac{27}{4}$

Exponentials & Logarithms

11. The curves $y = 4^x$ and $y = 3 \times 2^{x+1} - 8$ meet at exactly two points.
What is the gradient of the straight line through these two points?

A 2

B 4

C 6

D 8

E 12

F 16

Sequences & Series

12. An infinite geometric series has non-zero first term a and common ratio r

The sum to infinity of this series is S

The sum to infinity of the squares of the terms of this series is $2S$

The sum to infinity of the cubes of the terms of this series is $\frac{36}{7}S$

What is the value of S?

A 2

B 3

C 4

D 6

E 8

F 9

G 12

Trigonometry

13. A triangle has an angle of $120°$

The perimeter of the triangle is 15, and its area is $\frac{15\sqrt{3}}{4}$

What is the length of the longest side of the triangle?

A 5

B $\frac{11}{2}$

C 6

D $\frac{13}{2}$

E 7

F $\frac{15}{2}$

G 8

Integration

14. A curve has equation $y = 3\sqrt{x} - x$ for $x \geq 0$.

What is the total area enclosed between the curve, the x-axis, and the lines $x = 4$ and $x = 16$?

A $\frac{11}{2}$

B 8

C $\frac{27}{2}$

D 19

E $\frac{43}{2}$

F 27

Coordinate Geometry

15. A circle has equation

$$x^2 + y^2 - 2ax - 4y + 10 = 0$$

where a is a positive constant.

The two tangent lines to the circle that pass through the origin are perpendicular to each other.

What is the value of a?

A 1

B 2

C $2\sqrt{2}$

D 3

E $\sqrt{10}$

F 4

G $2\sqrt{6}$

Algebra

16. The straight line $y = k(x + 2)$, where k is a real constant, meets the graph $y = \dfrac{1}{x}$ at exactly one point.

What is the value of k?

A -2

B -1

C $-\dfrac{1}{2}$

D 0

E 1

Polynomials

17. The polynomial $P(x)$ is defined by

$$P(x) = 2x^4 + ax^3 + 3x^2 + bx + 6$$

where a and b are real constants.

The polynomial $x^2 - x + 2$ is a factor of $P(x)$.

What is the value of $a + b$?

A -7

B -3

C 3

D 7

E 11

Functions & Graphs

18. Define the function f for all real x by

$$f(x) = 2^x + 2^{-x}$$

For a given real number k, consider the equation $f(x) = k$.

For which values of k does this equation have exactly two distinct real solutions?

A $k < 2$

B $k = 2$

C $2 < k < 4$

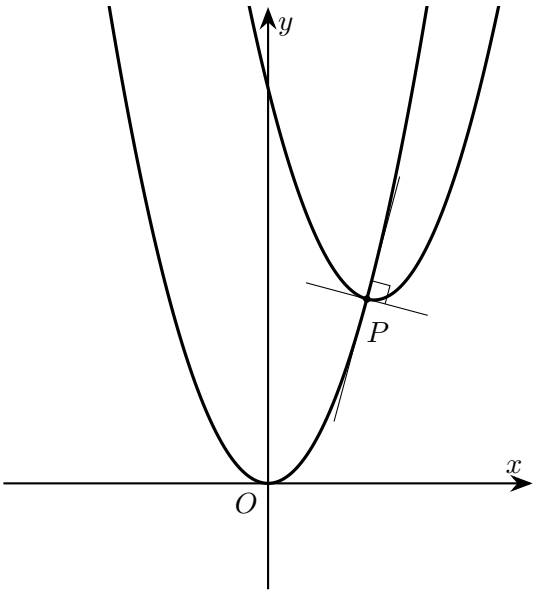
D $k = 4$

E $k > 2$

F $k \geq 2$

Differentiation

19. The diagram shows parts of the curves $y = x^2$ and $y = (x-2)^2 + b$, where b is positive.
The curves meet at P, and the tangents to the curves at P are perpendicular.
What is the value of b?



A $\frac{\sqrt{3}}{2}$

B $\sqrt{3}$

C $2\sqrt{3}$

D $3\sqrt{3}$

E $4\sqrt{3}$

Coordinate Geometry

20. Let C be the circle with equation

$$x^2 + y^2 = 25$$

A line ℓ has equation

$$y = mx + c$$

Which condition on c guarantees that ℓ is tangent to C?

A $c^2 = 25(1 - m^2)$

B $c^2 = 25(1 + m^2)$

C $c = 5(1 + m)$

D $|c| = 5 + m^2$

E $c^2 = 25m$

Differentiation

21. A curve has equation

$$y = \frac{(x+a)^2}{\sqrt{x}}, \qquad x > 0$$

where a is a real constant.

The tangent to the curve at $x = 4$ is parallel to the x-axis.

What is the sum of all possible values of a?

A -8

B 4

C 8

D 12

E 16

Coordinate Geometry

22. A circle has equation

$$x^2 + y^2 - 4x + 2y - 20 = 0$$

A chord of the circle has midpoint $(4, 1)$.

What is the length of the chord?

A $\sqrt{17}$

B $4\sqrt{2}$

C $2\sqrt{17}$

D $2\sqrt{21}$

E 10

F $2\sqrt{33}$

Sequences & Series

23. A geometric sequence has first term $\frac{3}{4}$ and common ratio r, where $0 < r < 1$.

The sum of all its terms from the second onwards equals the sum of the squares of all its terms.

What is the value of r?

A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{3}{4}$

Sequences & Series

24. In the expansion of $(1 + ax)^n$ in ascending powers of x, n is a positive integer and a is a real constant.

The first three terms of this expansion are $1 - 12x + 60x^2$.

What is the coefficient of x^3 in this expansion?

A -960

B -240

C -160

D -80

E 80

F 160

G 240

Trigonometry

25. Find the complete set of real values of k for which the equation

$$2\cos^2 x - k\sin x + k = 0$$

has exactly one solution in the range $0 \leq x < 2\pi$.

A $k < -4$ or $k > 0$

B $k \leq -4$ or $k > 0$

C $k < -4$ or $k \geq 0$

D $k \leq -4$ or $k \geq 0$

E $-4 < k < 0$

F $-4 \leq k \leq 0$

Functions & Graphs

26. The curve $y = f(x)$ has exactly one stationary point.

This stationary point is a minimum at $(4, -3)$.

Which of the following describes the stationary point of the curve

$$y = 5 - 2f(2x - 6)$$

A a maximum at $(5, 11)$

B a minimum at $(5, 11)$

C a maximum at $(-1, 11)$

D a maximum at $(2, 11)$

E a maximum at $(8, 11)$

F a maximum at $(5, -1)$

G a minimum at $(5, -1)$

Integration

27. The function $g(x)$ is defined for real x by

$$g(x) = |x - 1|$$

It is given that

$$\int_0^k g(x)\,\mathrm{d}x = 4$$

What is the value of k, with $k > 1$?

A $1 + 2\sqrt{2}$

B $\sqrt{7}$

C 3

D $1 + \sqrt{\frac{7}{2}}$

E $1 + \sqrt{7}$

F 4