ESAT-style practice paper

Set C Maths 1

Mathematics 1

TIME ALLOWED
40
MINUTES

27 multiple-choice questions

An average of 89 seconds per question.

Instructions

- Attempt all 27 questions.
- Calculators are *not* permitted.
- No formula booklet is provided.
- Use scrap paper or a whiteboard for rough working.

Information

- Each question has one correct answer and is worth one mark.
- No marks are deducted for incorrect answers.
- The questions are not arranged in order of difficulty.
- The ESAT itself is taken on a computer, with erasable sheets for rough working.

Questions

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Number

1. A map has scale $1 : 25\,000$.

A woodland covers an area of $12\,\text{cm}^2$ on the map.

What is the actual area of the woodland?

A $0.003\,\text{km}^2$

B $0.075\,\text{km}^2$

C $0.3\,\text{km}^2$

D $0.75\,\text{km}^2$

E $3\,\text{km}^2$

F $30\,\text{km}^2$

Coordinate Geometry

2. The straight line L_1 has equation $y = 2x + 4$.

The straight line L_2 is perpendicular to L_1 and passes through the point $P(1, 11)$.

The lines L_1 and L_2 intersect at the point Q.

The line L_1 crosses the y-axis at the point R.

What is the area of triangle PQR?

A $\frac{9}{2}$

B 5

C $\frac{15}{2}$

D 10

E 15

F $\frac{15\sqrt{5}}{2}$

G $\frac{45}{2}$

Algebra

3. The positive real numbers a, b and c satisfy

$$ab = 2 \qquad bc = 6 \qquad ca = 3$$

What is the value of $a^2 + b^2 + c^2$?

A 6

B 11

C 14

D 36

E 49

Algebra

4. Five parcels have a mean mass of 6 kg.

One parcel is replaced by a parcel with three times its mass, increasing the mean mass to 8 kg.

What is the mass of the parcel that was removed?

A 2 kg

B $\frac{10}{3}$ kg

C 5 kg

D 10 kg

E 15 kg

Number

5. Consider the integer

$$N = 8^{15} \times 5^{40}$$

When N is written out in full, how many digits does it have?

A 40

B 41

C 42

D 43

E 46

F 55

Number

6. Which one of the following is equal to

$$\frac{6}{3+\sqrt{3}}+\frac{4}{\sqrt{3}}-\sqrt{27}$$

A $3-\dfrac{26\sqrt{3}}{3}$

B $3-\dfrac{8\sqrt{3}}{3}$

C $3-\dfrac{2\sqrt{3}}{3}$

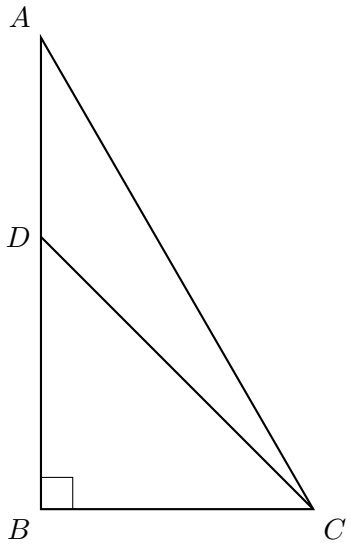
D $\dfrac{3}{2}-\dfrac{13\sqrt{3}}{6}$

E 3

F $3+\dfrac{10\sqrt{3}}{3}$

Trigonometry

7. In the diagram below, triangle ABC has a right angle at B.
The point D lies on AB.
Angle $BCD = 45°$ and angle $BCA = 60°$.
$CD = 6\sqrt{2}\,\text{cm}$.
What is the length of AD, in cm?



A $6\sqrt{3} - 6\sqrt{2}$

B $6 - 2\sqrt{3}$

C $6\sqrt{3} - 6$

D $12\sqrt{3} - 12$

E $6\sqrt{3}$

F $6\sqrt{3} + 6$

Coordinate Geometry

8. A point P has coordinates (x, y). The following transformations are applied in order:

1. reflection in the line $y = x$
2. rotation through $90°$ anticlockwise about the origin O
3. translation by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

The resulting point is Q. If P and Q are the same distance from O, which equation does P satisfy?

A $4x - 6y = 13$

B $4x + 6y = 13$

C $6x - 4y = 13$

D $6x + 4y = -13$

E $6x + 4y = 5$

F $6x + 4y = 13$

Algebra

9. The variables x, y and a are real, with $a \neq 0$.

They satisfy

$$y = 1 + \frac{(4-3x)^3}{2a}$$

Which of the following is a rearrangement of the formula to make x the subject?

A $x = \dfrac{4 - \sqrt[3]{\dfrac{y-1}{2a}}}{3}$

B $x = \dfrac{4 + \sqrt[3]{\dfrac{y-1}{2a}}}{3}$

C $x = \dfrac{4 - \sqrt[3]{2a(y-1)}}{3}$

D $x = \dfrac{4 + \sqrt[3]{2a(y-1)}}{3}$

E $x = \dfrac{4 - \sqrt[3]{2a(y+1)}}{3}$

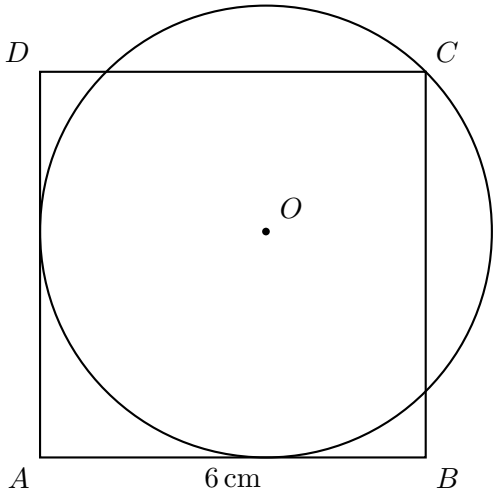
F $x = \dfrac{4 + \sqrt[3]{2a(y+1)}}{3}$

Geometry

10. $ABCD$ is a square with side length 6 cm.

A circle touches the sides AB and AD and passes through C. Its centre lies inside the square.

What is the radius of the circle, in cm?



A $6(\sqrt{2}-1)$

B $6(2-\sqrt{2})$

C $3\sqrt{2}$

D $12(\sqrt{2}-1)$

E $6(2+\sqrt{2})$

Geometry

11. $ABCD$ is a trapezium with AB parallel to CD.
The diagonals AC and BD meet at P.
The area of triangle APB is $18\,\text{cm}^2$, and the area of triangle CPD is $8\,\text{cm}^2$.
What is the area of $ABCD$?

A $26\,\text{cm}^2$

B $38\,\text{cm}^2$

C $42\,\text{cm}^2$

D $44\,\text{cm}^2$

E $50\,\text{cm}^2$

F $54\,\text{cm}^2$

Sequences & Series

12. A student attempts to write down the first six terms of a sequence whose nth term is an^2+bn+c, where a, b and c are constants and $a \neq 0$.

The student writes the following six numbers, in this order:

$$4, \quad 13, \quad 27, \quad 43, \quad 64, \quad 89$$

Exactly one of these six numbers is incorrect.

Which of the following statements is true?

A The 1st number should be 3.

B The 2nd number should be 14.

C The 3rd number should be 26.

D The 3rd number should be 28.

E The 4th number should be 46.

F The 5th number should be 63.

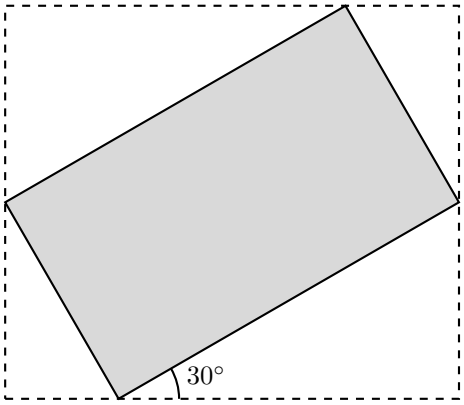
G The 6th number should be 90.

Trigonometry

13. A rectangular card is placed with its longer sides at 30° to the horizontal.

The smallest rectangle with horizontal and vertical sides that contains the card has twice the area of the card, as shown.

What is the ratio of the longer side of the card to its shorter side?



A $\sqrt{2} : 1$

B $\sqrt{3} : 1$

C $2 : 1$

D $\dfrac{4\sqrt{3}}{3} : 1$

E $(2 + \sqrt{3}) : 1$

Geometry

14. A sector of a circle of radius r has angle $\theta°$ at the centre, where $0° < \theta < 360°$

The perimeter of the sector consists of the two straight radii together with the arc. This perimeter is equal to the circumference of the whole circle.

What fraction of the area of the whole circle is the area of the sector?

A $\dfrac{1}{\pi}$

B $\dfrac{2}{\pi}$

C $\dfrac{1}{2} - \dfrac{1}{\pi}$

D $1 - \dfrac{2}{\pi}$

E $1 - \dfrac{1}{\pi}$

F $\dfrac{\pi - 1}{\pi + 1}$

G $\dfrac{\pi}{\pi + 1}$

Polynomials

15. The curve $y = ax^2 + bx + c$, where a, b and c are constants, has a turning point at $(3, -8)$.
The curve passes through the point $(0, 10)$.
At which values of x does the curve cross the x-axis?

A $x = 1$ and $x = 5$

B $x = -5$ and $x = -1$

C $x = -1$ and $x = 7$

D $x = 3 - 2\sqrt{2}$ and $x = 3 + 2\sqrt{2}$

E $x = 3 - \sqrt{2}$ and $x = 3 + \sqrt{2}$

F The curve does not cross the x-axis.

Coordinate Geometry

16. The line $y = x - 2$ meets the curve $y = \dfrac{8}{x}$ at two points, P and Q.

What is the area of triangle OPQ, where O is the origin?

A 3

B 6

C $6\sqrt{2}$

D 12

E $12\sqrt{2}$

F 24

Geometry

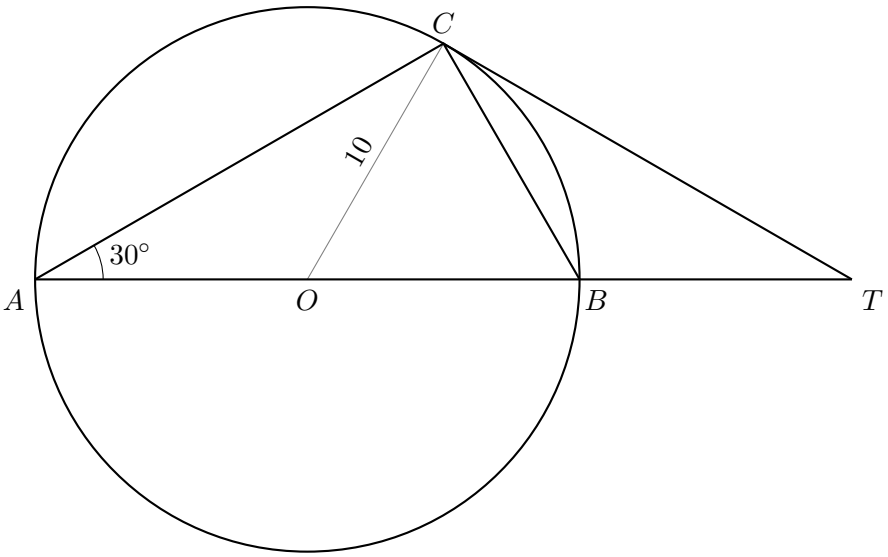
17. A circle has centre O and radius 10.

The points A and B lie on the circle, and AB is a diameter.

The point C lies on the circle, and the angle CAB is 30°.

The tangent to the circle at C meets the line AB at the point T.

What is the area of triangle BCT?



A $\dfrac{25\sqrt{3}}{2}$

B $25\sqrt{3}$

C $\dfrac{75\sqrt{3}}{2}$

D $50\sqrt{3}$

E $75\sqrt{3}$

F $100\sqrt{3}$

Probability & Counting

18. Eight cards are numbered from 1 to 8.

Two cards are drawn at random, without replacement.

The product of the two numbers is not divisible by 3.

What is the probability that their sum is divisible by 3?

A $\frac{1}{3}$

B $\frac{2}{5}$

C $\frac{1}{2}$

D $\frac{3}{5}$

E $\frac{2}{3}$

Algebra

19. x and y are real numbers.

What is the smallest possible value of $x^2 - 6x + y^2 + 2y + 15$?

A -10

B 4

C 5

D 6

E 14

F 25

Algebra

20. Three cogwheels, A, B and C, are arranged in a row so that A meshes directly with B, and B meshes directly with C; A does not touch C.

Wheel A has 12 teeth and turns at 300 rpm clockwise. Wheel B has 18 teeth. Wheel C has 24 teeth.

What is the rotation speed and direction of wheel C?

A 150 rpm, clockwise

B 150 rpm, anticlockwise

C 200 rpm, clockwise

D 200 rpm, anticlockwise

E 600 rpm, clockwise

F 600 rpm, anticlockwise

Geometry

21. A walker starts at the point P.

She walks 6 km on a bearing of 040°, and then walks 6 km on a bearing of 130°, finishing at the point Q.

What is the distance PQ, and what is the bearing of P from Q?

A $6\sqrt{2}$ km, bearing 085°

B $6\sqrt{2}$ km, bearing 220°

C $6\sqrt{2}$ km, bearing 265°

D $6\sqrt{2}$ km, bearing 310°

E $6\sqrt{3}$ km, bearing 265°

F 12 km, bearing 085°

G 12 km, bearing 265°

Algebra

22. The real numbers x and y are non-zero, and $x \neq y$.

A student subtracts two fractions by subtracting the numerators and subtracting the denominators, and writes

$$\frac{3}{x} - \frac{4}{y} = \frac{3-4}{x-y}$$

This statement is false for most values of x and y, but it is true for some values of x and y.

Which of the following gives all the possible values of $\frac{x}{y}$ for which the statement is true?

A $-\frac{3}{2}$ or $\frac{1}{2}$

B $-\frac{1}{2}$ or $\frac{3}{2}$

C $\frac{1}{2}$ or $\frac{3}{2}$

D $\frac{2}{3}$ or 2

E $\frac{3}{4}$ or 1

F $\frac{3}{2}$ only

G There are no possible values.

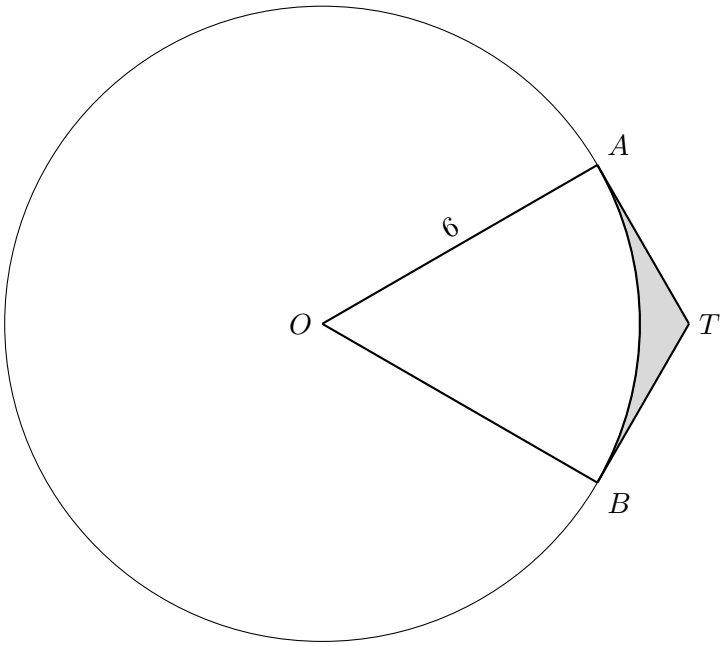
Geometry

23. A circle has centre O and radius 6 cm.

The chord AB also has length 6 cm.

The tangents to the circle at A and B meet at T.

What is the area, in cm^2, enclosed by the line segments TA and TB and the minor arc AB?



A $6\pi - 9\sqrt{3}$

B $12\sqrt{3} - 6\pi$

C $6\pi - 3\sqrt{3}$

D $12\sqrt{3} - 3\pi$

E $12\sqrt{3}$

Number

24. Liquid P has density $0.6\,\mathrm{g\,cm^{-3}}$.
Liquid Q has density $1.5\,\mathrm{g\,cm^{-3}}$.
A mixture is made from P and Q in the ratio $3:5$ by mass.
The volume of the mixture is equal to the total volume of the liquids used.
What is the density of the mixture in $\mathrm{kg\,m^{-3}}$?

A 0.96

B 96

C 960

D 1050

E 1162.5

F 9600

Algebra

25. w, x and y are positive variables.

w is directly proportional to x and inversely proportional to y.

The value of x decreases by 20%, and the value of y is halved.

What is the percentage increase in w?

A 20%

B 40%

C 60%

D 100%

E 160%

Polynomials

26. The quadratic graph $y = x^2 + bx + c$, where b and c are real constants, crosses the x-axis at two points P and Q.

The distance PQ is 6.

The graph crosses the y-axis at the point $(0, 7)$.

What is the y-coordinate of the turning point of the graph?

A -36

B -9

C -3

D 3

E 7

F It cannot be determined from the information given

Probability & Counting

27. Two fair spinners each have six equal sectors, labelled 1, 2, 3, 4, 5 and 6.

Each spinner is spun once.

It is known that at least one spinner shows an even number.

What is the probability that the larger number is exactly twice the smaller number?

A $\frac{1}{12}$

B $\frac{1}{9}$

C $\frac{1}{6}$

D $\frac{2}{9}$

E $\frac{1}{3}$