ESAT-style practice paper

Set B Maths 2

Mathematics 2

TIME ALLOWED
40
MINUTES

27 multiple-choice questions

An average of 89 seconds per question.

Instructions

- Attempt all 27 questions.
- Calculators are *not* permitted.
- No formula booklet is provided.
- Use scrap paper or a whiteboard for rough working.

Information

- Each question has one correct answer and is worth one mark.
- No marks are deducted for incorrect answers.
- The questions are not arranged in order of difficulty.
- The ESAT itself is taken on a computer, with erasable sheets for rough working.

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Algebra

1. The equation

$$(x^2 - 3x)^2 = 4(x^2 - 3x) + 12$$

has four real solutions.

What is the sum of all four solutions?

A 0

B 2

C 3

D 6

E 9

F 12

Inequalities

2. The real number x satisfies both of the following inequalities.

$$3x^2 + 5x \geq 2 \qquad \text{and} \qquad x^2 < 2x + 8$$

What is the complete set of possible values of x?

A $\frac{1}{3} \leq x < 4$

B $\frac{1}{3} < x < 4$

C $x = -2$ or $\frac{1}{3} \leq x < 4$

D $-2 < x \leq \frac{1}{3}$

E $-2 < x \leq -\frac{1}{3}$ or $2 \leq x < 4$

F $x \leq -2$ or $x \geq \frac{1}{3}$

G $-2 < x < 4$

Trigonometry

3. In triangle ABC,

$$AB = 6 \text{ cm}, \qquad AC = 4 \text{ cm}, \qquad \text{angle } ABC = 30^\circ$$

What is the sum of the two possible values of BC?

A $2\sqrt{7}$ cm

B $3\sqrt{3}$ cm

C $4\sqrt{3}$ cm

D $6\sqrt{3}$ cm

E 8 cm

F 20 cm

Sequences & Series

4. In the binomial expansion of $(1 + kx)^7$, where k is a constant with $0 < k < 1$, the coefficients of x, x^2 and x^3 are the first three terms, in that order, of an arithmetic progression.

What is the sum of the first 11 terms of this arithmetic progression?

A $-\frac{231}{5}$

B $-\frac{154}{5}$

C $-\frac{77}{5}$

D $-\frac{21}{5}$

E $\frac{77}{5}$

F $\frac{231}{5}$

Integration

5. A curve is defined for $x > 0$ and passes through the point $(1, 0)$.

The gradient of the curve is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x - 4}{\sqrt{x}}$$

The curve has exactly one stationary point.

What is the y-coordinate of this stationary point?

A -18

B $-\frac{32}{3}$

C $-\frac{10}{3}$

D $\frac{8}{3}$

E $\frac{17}{2}$

F 18

Coordinate Geometry

6. A point P lies on the circle

$$x^2 + y^2 = 25$$

What is the smallest possible value of the distance from P to the point $(8, 3)$?

A 3

B $3\sqrt{2}$

C $4\sqrt{3} + 5$

D $\sqrt{73} - 5$

E $\sqrt{73}$

F $\sqrt{73} + 5$

G $5 - \sqrt{73}$

Sequences & Series

7. A geometric series has first term a and common ratio r, where $a \neq 0$.

The sum to infinity of the series is 4.

The squares of the terms of this series form a second geometric series, whose sum to infinity is $\frac{48}{5}$.

What is the second term of the original series?

A $-\frac{5}{4}$

B $\frac{1}{4}$

C $\frac{9}{16}$

D $\frac{3}{4}$

E 1

F $\frac{5}{4}$

Exponentials & Logarithms

8. The positive real numbers x and y satisfy the simultaneous equations

$$\begin{aligned} 2\log_3 x - \log_3 y &= 1 \\ \log_3(2y - x) &= 2 \end{aligned}$$

What is the value of y?

A $\frac{9}{4}$

B 3

C $\frac{9}{2}$

D 6

E $\frac{27}{4}$

F $\frac{27}{2}$

Coordinate Geometry

9. The point A has coordinates $\left(3, \frac{1}{4}\right)$. Two distinct lines through A are tangent to the curve C with equation

$$y = \frac{1}{x}, \qquad x \neq 0.$$

Let the gradients of these tangent lines be p and q. What is the value of pq?

A $\frac{1}{144}$

B $\frac{1}{40}$

C $\frac{1}{16}$

D $\frac{1}{12}$

E $\frac{1}{9}$

F $\frac{5}{18}$

G -1

H 0

Differentiation

10. The curve C has equation

$$y = x^4 - 4x^3 + ax^2$$

where a is a real constant.

Find the complete set of values of a for which C has exactly three stationary points.

A $a < 0$

B $0 < a < \frac{9}{2}$

C $a < \frac{9}{2}$

D $a < \frac{9}{2}$ and $a \neq 0$

E $a \leq \frac{9}{2}$ and $a \neq 0$

F $a > \frac{9}{2}$

G all real values of a except $a = \frac{9}{2}$

Sequences & Series

11. An infinite geometric sequence has real terms and common ratio r, where $|r| < 1$.

The sum of its first four terms is 1.

The sum of its fifth to twelfth terms, inclusive, is $\frac{3}{4}$.

What is the sum of all the terms of the sequence?

A $\frac{2}{5}$

B $\frac{8}{5}$

C $\frac{7}{4}$

D 2

E 4

Functions & Graphs

12. The graph of $y = h(x)$ is obtained from the graph of $y = \sqrt{9 - x^2}$ by first stretching by a factor of 2 parallel to the y-axis, then translating 1 unit to the right, and finally translating 2 units down

Which equation defines $h(x)$?

A $y = 2\sqrt{9 - x^2} - 2$

B $y = \sqrt{9 - (2x - 1)^2} - 2$

C $y = 2\sqrt{9 - (x + 1)^2} - 2$

D $y = 2\sqrt{9 - (x - 1)^2} - 2$

E $y = 2\sqrt{9 - (x - 1)^2 + 2}$

Coordinate Geometry

13. A circle has equation

$$x^2 + y^2 - 8x + 2y - 8 = 0$$

The line $y = x + c$, where c is a constant, meets the circle at two points A and B.

The length of AB is 8.

What are the possible values of c?

A $c = -2$ or $c = -8$

B $c = -5 + \frac{3\sqrt{2}}{2}$ or $c = -5 - \frac{3\sqrt{2}}{2}$

C $c = -5 + 3\sqrt{2}$ or $c = -5 - 3\sqrt{2}$

D $c = -5 + 4\sqrt{2}$ or $c = -5 - 4\sqrt{2}$

E $c = 5 + 3\sqrt{2}$ or $c = 5 - 3\sqrt{2}$

F $c = 8$ or $c = 2$

Algebra

14. The line $y = 2x + 1$ meets the curve $x^2 + xy + y^2 = 13$ at two points, A and B. What is the length of AB?

A $\frac{5\sqrt{5}}{7}$

B $\frac{19}{7}$

C $\frac{19\sqrt{5}}{14}$

D $\frac{19\sqrt{3}}{7}$

E $\frac{38}{7}$

F $\frac{19\sqrt{5}}{7}$

Functions & Graphs

15. Find the complete set of values of k for which the equation

$$|x^3 - 12x| = k$$

has exactly four distinct real solutions.

A There are no such values of k

B $k = 0$ only

C $k = 16$ only

D $k = 32$ only

E $0 < k < 16$

F $k > 16$

G $k = 0$ or $k = 16$

Sequences & Series

16. A sequence a_n is defined by $a_0 = 3$ and, for $n \geq 0$,

$$a_{n+1} = 3(a_n)^2$$

Which of the following is equal to $\log_3(a_n)$?

A 2^n

B $2^n - 1$

C $2^{n+1} - 1$

D $2^{n+1} + 1$

E $2^n - 2$

F $2^{n+1} - 2$

Coordinate Geometry

17. A circle has equation

$$x^2 + y^2 - 6x + 4y - 12 = 0$$

The point P on the circle has coordinates $(6, 2)$.

What is the equation of the tangent to the circle at P?

A $3x + y = 20$

B $3x - 4y = 10$

C $3x + 4y = 26$

D $4x - 3y = 18$

E $4x + 3y = 30$

Differentiation

18. The curve C has equation $y = x^3 - 3x$.

There are two points on C at which the tangent has gradient 9.

What is the distance between the points where these two tangents meet the y-axis?

A 16

B 24

C 32

D 36

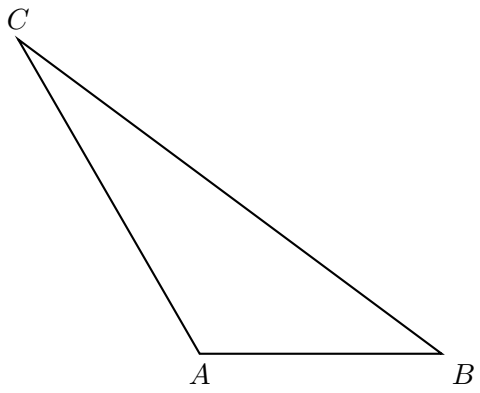
E 48

Trigonometry

19. A triangle ABC has $AB = 4$ and $AC = 6$.

The area of the triangle is $6\sqrt{3}$, and angle BAC is obtuse.

What is the length of BC?



A $\sqrt{52 - 24\sqrt{3}}$

B $2\sqrt{7}$

C $2\sqrt{13}$

D $2\sqrt{19}$

E $\sqrt{52 + 24\sqrt{3}}$

Geometry

20. Let C be the circle $x^2 + y^2 = 9$

Let P be the point $(a, 0)$ where $a > 3$

The two tangents from P to C touch the circle at T_1 and T_2

What is the area of triangle PT_1T_2 in terms of a?

A $3\sqrt{a^2 - 9}$

B $\dfrac{3(a^2 - 9)^{3/2}}{a^2}$

C $\dfrac{9}{a}\sqrt{a^2 - 9}$

D $\dfrac{a}{3}\sqrt{a^2 - 9}$

E $\dfrac{a^2 - 9}{3}$

F $\dfrac{a^2 - 9}{a}$

G $\dfrac{9}{\sqrt{a^2 - 9}}$

Trigonometry

21. The function f is defined for all real x by

$$f(x) = \sin^2 x + 2\cos x$$

How many solutions does the equation $f(x) = k$ have in the interval $0 \leq x \leq 2\pi$, where k is the real constant chosen so that k is the value of $f(x)$ at $x = \frac{2\pi}{3}$?

A 1

B 2

C 3

D 4

E 5

F 6

Integration

22. The curve C has equation $y = \sqrt{x}\,(a - x)$ for $x \geq 0$, where a is a positive constant.

The finite region enclosed by C and the x-axis has area $\dfrac{128}{15}$.

What is the value of a?

A 2

B $2^{6/5}$

C $2^{8/5}$

D 4

E $2^{12/5}$

F 8

G $2^{10/3}$

Differentiation

23. A curve has equation $y = 6\sqrt{x} - x$, for $x > 0$.

The point P on the curve has x-coordinate 4.

The tangent to the curve at P meets the x-axis at A.

The normal to the curve at P meets the x-axis at B.

What is the area of triangle APB?

A 20

B 40

C $\frac{208}{3}$

D 80

E 128

F 160

Polynomials

24. Let k be an integer with $0 \leq k \leq 10$

Consider the quadratic equation

$$x^2 - 2kx + (k^2 - 3k + 2) = 0$$

For how many values of k does the equation have two distinct positive real roots?

A 6

B 7

C 8

D 9

E 10

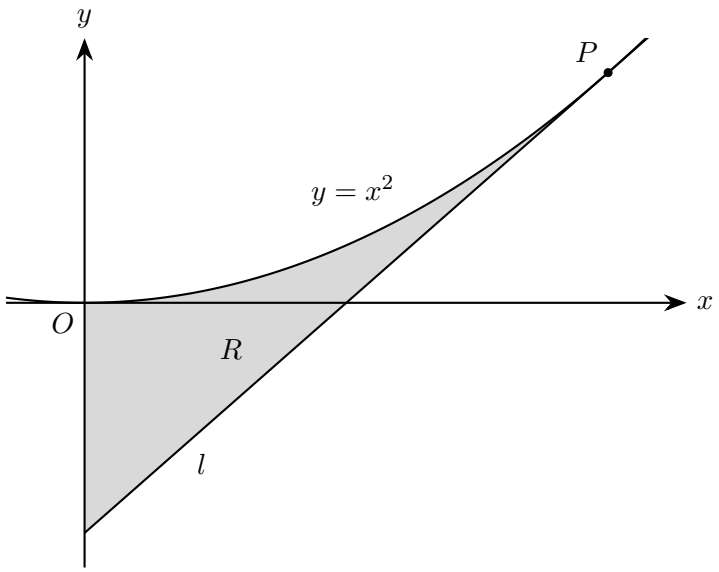
Integration

25. The diagram shows the curve $y = x^2$, where $a > 0$.

The line l is the tangent to the curve at P, where P has coordinates (a, a^2).

The curve, the line l and the y-axis enclose a region R.

What fraction of the area of R lies below the x-axis?



A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{3}{4}$

Polynomials

26. Consider

$$P(x) = x^4 + ax^3 + bx^2 + ax + 1$$

where a and b are real constants.

Suppose $(x-2)$ and $\left(x+\frac{1}{2}\right)$ are both factors of $P(x)$

What is the value of b?

A -13

B -5

C 0

D 5

E 13

F $-\frac{17}{4}$

Exponentials & Logarithms

27. Consider the equation

$$2^{2x+1} - 7 \times 2^x + 3 = 0$$

What is the sum of all the real solutions of this equation?

A $\frac{3}{2}$

B $\log_2\left(\frac{3}{4}\right)$

C $\log_2\left(\frac{3}{2}\right)$

D $\log_2 3$

E $\log_2\left(\frac{7}{2}\right)$

F $\log_2 6$