ESAT-style practice paper

Set A Maths 2

Mathematics 2

TIME ALLOWED
40
MINUTES

27 multiple-choice questions

An average of 89 seconds per question.

Instructions

- Attempt all 27 questions.
- Calculators are *not* permitted.
- No formula booklet is provided.
- Use scrap paper or a whiteboard for rough working.

Information

- Each question has one correct answer and is worth one mark.
- No marks are deducted for incorrect answers.
- The questions are not arranged in order of difficulty.
- The ESAT itself is taken on a computer, with erasable sheets for rough working.

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Algebra

1. The positive number x satisfies

$$\frac{(27x^3)^{\frac{2}{3}}}{(16x^2)^{\frac{1}{4}}} = 6\sqrt{x}$$

What is the value of x?

A $\frac{2}{3}$

B $\frac{3}{4}$

C $\frac{4}{3}$

D $\frac{3}{2}$

E $\frac{16}{9}$

F $\frac{8}{3}$

Exponentials & Logarithms

2. Consider the equation

$$2^{x+2} + 2^{3-x} = 33$$

where x is real.

The equation has two real solutions.

What is the product of these two solutions?

A -6

B -2

C -1

D 1

E 2

F 6

Integration

3. Evaluate

$$\int_1^4 \left(\frac{2}{x}+\sqrt{x}\right)^2 \mathrm{d}x - \int_1^4 \left(\frac{2}{x}-\sqrt{x}\right)^2 \mathrm{d}x$$

A 0

B 4

C 8

D 12

E 16

F 20

G 24

H 28

Differentiation

4. The curve C has equation $y = x^3 - 3x^2 - 4x + 2$

The tangent to C at the point where $x = 2$ meets C again at exactly one further point, where $x = a$. What is the value of a?

A -7

B -4

C -1

D 1

E 2

F 5

Coordinate Geometry

5. For a positive constant c, the line $y = x + c$ meets the graph $y = \dfrac{1}{x}$ at points A and B

The distance AB is $6\sqrt{2}$

What is the value of c?

A $\sqrt{14}$

B $4\sqrt{2}$

C 6

D $2\sqrt{10}$

E $2\sqrt{17}$

F $6\sqrt{2}$

Integration

6. Let

$$F(x) = \int_0^x (t^2 - 4t + 1)\,\mathrm{d}t$$

For how many real values of x does $F(x) = 0$ hold?

A 0

B 1

C 2

D 3

E 4

F 5

Sequences & Series

7. A geometric progression has a real first term and a real common ratio.
The sum of the first three terms is 14, and the sum of the first six terms is 126.
What is the sum of the first nine terms?

A 98

B 252

C 511

D 1022

E 1092

F 1134

Polynomials

8. Determine the number of stationary points on the curve with equation

$$y = x^4 - 6x^2 + 4x$$

A 0

B 1

C 2

D 3

E 4

F 5

Exponentials & Logarithms

9. For $0 < x < 6$, the equation

$$\log_2 x + \log_2(6 - x) = 3$$

has two solutions. What is the sum of these two solutions?

A 2

B 3

C 4

D 6

E 8

Coordinate Geometry

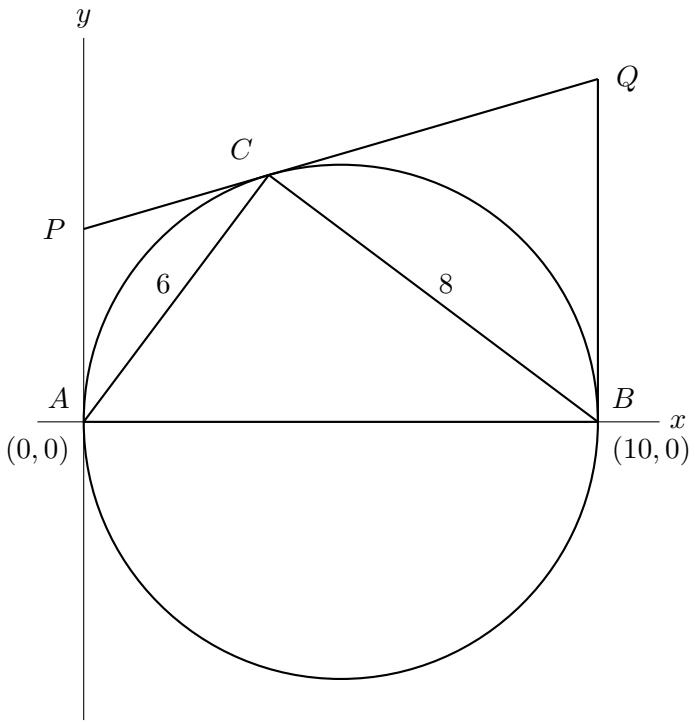
10. The points A and B have coordinates $(0, 0)$ and $(10, 0)$ respectively.

A circle has diameter AB.

The point C lies on the circle above the x-axis, with $AC = 6$ and $BC = 8$.

The tangent to the circle at C meets the lines $x = 0$ and $x = 10$ at P and Q respectively.

What is the length PQ?



A $\frac{35}{12}$

B $\frac{15}{4}$

C $\frac{20}{3}$

D 10

E $\frac{125}{12}$

F $\frac{125}{6}$

Trigonometry

11. In triangle ABC, angle $BAC = 30°$, $AC = 5$ cm and $BC = 3$ cm.
There are two possible triangles satisfying these conditions, giving two possible lengths for AB.
What is the sum, in cm, of the two possible values of AB?

A 5

B $\sqrt{11}$

C $4\sqrt{3}$

D $5\sqrt{3}$

E $\frac{5\sqrt{3}+\sqrt{11}}{2}$

F $10\sqrt{3}$

Sequences & Series

12. An infinite geometric series has positive first term a and common ratio r, where $|r| < 1$. The sum to infinity of this series is S.

A second series is formed by taking every third term of the first series, beginning with the first term:

$$T = u_1 + u_4 + u_7 + u_{10} + \dots$$

where u_n denotes the nth term of the first series.

What is the maximum possible value of $\dfrac{T}{S}$?

A $\dfrac{3}{4}$

B 1

C $\dfrac{7}{6}$

D $\dfrac{4}{3}$

E $\dfrac{3}{2}$

F 3

Trigonometry

13. How many distinct real values of x satisfy the equation

$$\cos(2x) = 1 - x$$

in the range $0 \leq x \leq \pi$?

A 0

B 1

C 2

D 3

E 4

F 5

G 6

Differentiation

14. A curve has equation

$$y = \frac{8}{\sqrt{x}}, \qquad x > 0$$

The point P on the curve has x-coordinate 4.

The tangent to the curve at P meets the x-axis at A.

The normal to the curve at P meets the x-axis at B.

What is the area of triangle PAB?

A 10

B 16

C 20

D 34

E 36

F 40

Coordinate Geometry

15. Let P be a point in the (x, y)-plane such that the distance from P to $(2, 0)$ is twice the distance from P to $(-3, 0)$

The set of possible values of P is a circle.

What is the radius of this circle?

A $\frac{5}{3}$

B $\frac{5}{2}$

C 3

D $\frac{10}{3}$

E $\frac{7}{2}$

F 4

G $\frac{20}{3}$

Sequences & Series

16. The nth term of a sequence is a quadratic expression in n.
The second, fifth and eighth terms are 4, 13 and 40, respectively.
What is the eleventh term?

A 67

B 76

C 85

D 94

E 121

Differentiation

17. The function f is defined for $x > 0$ by

$$f(x) = 2x^{\frac{3}{2}} - 9x + 12\sqrt{x}$$

What is the complete set of values of x for which $f'(x) < 0$?

A $1 < x < \sqrt{2}$

B $1 < x < 2$

C $1 < x < 4$

D $1 < x < 16$

E $0 < x < 1$ or $x > 2$

F $0 < x < 1$ or $x > 4$

Exponentials & Logarithms

18. The real numbers x and y satisfy the simultaneous equations

$$\begin{aligned} 2\log_2(x-5) - \log_2 y &= 2 \\ \log_2 y - \log_2(x+7) &= -1 \end{aligned}$$

What is the value of $x + y$?

A 5

B 10

C 13

D 15

E 17

F 20

Sequences & Series

19. A geometric series has first term a and common ratio r, where $a \neq 0$.

The sum to infinity of this series is 8.

A second series is formed by squaring each term of the original series.

The sum to infinity of the second series is 32.

What is the common ratio of the original series?

A $-\frac{1}{3}$

B $\frac{1}{9}$

C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{1}{2}$

F $\frac{2}{3}$

Integration

20. The curve $y = 4\sqrt{x}$ and the line $y = x + 3$ meet at two points.

What is the area of the finite region enclosed by the curve and the line?

A $\frac{11}{3}$

B $\frac{9}{2}$

C $\frac{16}{3}$

D $\frac{32}{3}$

E 12

F 64

Polynomials

21. Find the complete set of real values of k for which the quadratic equation

$$x^2 - (k+2)x + (2k+1) = 0$$

has two distinct **positive** real roots.

A $k > 4$

B $k < 0$ or $k > 4$

C $-\frac{1}{2} < k < 0$ or $k > 4$

D $-2 < k < 0$ or $k > 4$

E $k > -\frac{1}{2}$

Coordinate Geometry

22. A circle has equation $x^2 + y^2 = 25$

From the point $(7, 1)$ two tangents to the circle are drawn with gradients m_1 and m_2

What is the value of $m_1 m_2$?

A -1

B $-\frac{3}{4}$

C 0

D 1

E $\frac{4}{3}$

Inequalities

23. The curve C has equation $y = x^2 + kx + 2$, where k is a real constant.

The line ℓ has equation $y = x - k$.

The curve C and the line ℓ do not meet.

What is the complete set of possible values of k?

A $-1 < k < 7$

B $-1 \leq k \leq 7$

C $k < -1$ or $k > 7$

D $-7 < k < 1$

E $k < -7$ or $k > 1$

F $1 < k < 7$

G $k < 7$

Sequences & Series

24. The constant a is a real number.

In the expansion of

$$(1 + ax)(2 - x)^6$$

in ascending powers of x, the coefficient of x^2 is zero.

What is the value of a?

A $-\frac{5}{4}$

B $\frac{5}{16}$

C $\frac{5}{8}$

D $\frac{4}{5}$

E $\frac{5}{4}$

F $\frac{5}{2}$

Trigonometry

25. In triangle ABC, $AB = 8\,\text{cm}$ and $AC = 5\,\text{cm}$.
The area of triangle ABC is $10\sqrt{3}\,\text{cm}^2$.
Angle BAC is obtuse.
What is the length of BC, in cm?

A 7

B $\sqrt{69}$

C $\sqrt{89}$

D $\sqrt{109}$

E $\sqrt{129}$

F 13

Differentiation

26. The curve with equation $y = \dfrac{1}{x}$ is defined for $x \neq 0$.

A point P lies on this curve in the first quadrant.

The tangent to the curve at P crosses the x-axis and the y-axis, and together with the coordinate axes it encloses a triangle.

What is the area of this triangle?

A $\dfrac{1}{2}$

B 1

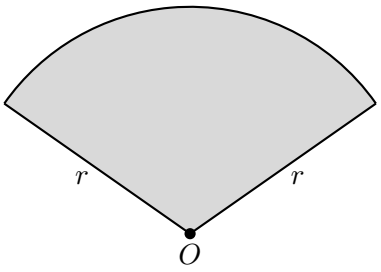
C 2

D 4

E It depends on the position of P

Geometry

27. A sector of a circle is bounded by two radii and an arc.
The total length of the boundary of the sector is 40 cm.
What is the greatest possible area of the sector?



A 50 cm^2

B 100 cm^2

C $\frac{400}{\pi}$ cm^2

D 200 cm^2

E 400 cm^2

F 100π cm^2