1. Let

$$f(x) = a^{\cos x} + a^{-\cos x}$$

with $a > 0$ and real x

The difference between the maximum and minimum values of $f(x)$ is 4

Find the sum of the possible values of a

A 2

B 3

C 4

D 5

E 6

F $3\sqrt{2}$

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2. For non-zero real numbers a and b, consider the equation

$$\log_3(x - a) = \log_3 x + b$$

The number of real solutions x depends on a and b.

Which of the following is correct?

A Exactly one real solution if $a(1 - 3^b) > 0$ **and** no real solutions if $a(1 - 3^b) < 0$

B Exactly one real solution if $a(1 - 3^b) < 0$ **and** no real solutions if $a(1 - 3^b) > 0$

C Exactly two real solutions if $a(1 - 3^b) > 0$ **and** none otherwise

D No real solutions if $ab > 0$ **and** exactly one real solution if $ab < 0$

E The equation always has exactly one real solution for all non-zero a and b

3. Fix $a > 0$ with $a \neq 1$

For $x > 0$ consider the equation

$$\log_a x = x$$

The number of positive solutions of this equation is

A exactly one if $0 < a < 1$, two if $1 < a < e^{1/e}$, one if $a = e^{1/e}$, and none if $a > e^{1/e}$

B exactly two if $0 < a < 1$, one if $a = 1$, and none if $a > 1$

C exactly one for all $a > 0$, $a \neq 1$

D none if $0 < a < 1$, one if $a = e^{1/e}$, and two if $a > e^{1/e}$

E none if $a > 1$, and exactly one if $0 < a < 1$

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4. Define functions g_1, g_2, g_3, g_4 on the real numbers by

$$\begin{aligned} g_1(x) &= \sin x \\ g_2(x) &= \sin(\sin x) \\ g_3(x) &= \sin(\sin(\sin x)) \\ g_4(x) &= \sin(\sin(\sin(\sin x))) \end{aligned}$$

Let M_k be the maximum value of g_k for $k = 1, 2, 3, 4$

Which one of the following statements is true?

A $M_1 = M_2 = M_3 = M_4 = 1$

B $1 > M_1 > M_2 > M_3 > M_4 > 0$

C $1 > M_1 = M_2 = M_3 = M_4 > 0$

D $M_1 = 1$ and $1 > M_2 > M_3 > M_4 > 0$

E $M_2 = M_4$ and $M_1 = M_3$

F None of the above

5. Consider

$$P(x) = (1+x)^8(1-2x)^5$$

The coefficient of x^3 in $P(x)$ is equal to k times the coefficient of x^2 in $P(x)$

What is the value of k?

A $-\dfrac{9}{4}$

B $-\dfrac{1}{4}$

C 0

D $\dfrac{1}{4}$

E $\dfrac{9}{4}$

F $-\dfrac{4}{3}$

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6. Let f be a polynomial defined for all real x, and define

$$J = \int_{-2}^{3} \big(f(x) - f(|x|)\big)\,\mathrm{d}x$$

Which of the following expressions is equal to J for all such polynomials f?

A $-\displaystyle\int_0^2 f(t)\,\mathrm{d}t$

B $\displaystyle\int_0^2 f(t)\,\mathrm{d}t$

C $\displaystyle\int_{-2}^0 f(t)\,\mathrm{d}t - \int_0^2 f(t)\,\mathrm{d}t$

D 0

E $f(0)$

F $f(2) - f(0)$

G $\displaystyle\int_{-2}^0 f(t)\,\mathrm{d}t$

7. Let $a > 0$ and k be real numbers

Consider the cubic

$$g(x) = x^3 - 3ax + k$$

For which complete range of values of k does the equation $g(x) = 0$ have three distinct real roots?

A $-2a^{3/2} < k < 2a^{3/2}$

B $-2a^{3/2} \leq k \leq 2a^{3/2}$

C $-a^{3/2} < k < a^{3/2}$

D $-2a^3 < k < 2a^3$

E $k < -2a^{3/2}$ **or** $k > 2a^{3/2}$

F $-2a^{3/2} < k \leq 2a^{3/2}$

G $-2a^{3/2} \leq k < 2a^{3/2}$

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8. Consider the curve defined by

$$(|x| - 4)^2 + (y - 1)^2 = 1$$

How many distinct straight lines passing through the origin are tangents to this curve?

A 0

B 1

C 2

D 3

E 4

F 5

9. For a real constant k, define

$$A(k) = \int_{-1}^{2} |x - k| \, \mathrm{d}x$$

For which value of k is $A(k)$ minimised?

A -1

B $\frac{1}{2}$

C 1

D $\frac{3}{2}$

E 2

F 0

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10. Let a and b be real numbers with $a + b = 1$

What is the least possible value of

$$(a^2 + b)(b^2 + a) \quad ?$$

A $\frac{9}{16}$

B $\frac{1}{4}$

C $\frac{1}{2}$

D $\frac{3}{4}$

E 1

11. Let k be an integer with $0 \leq k \leq 10$

Consider the quadratic equation

$$x^2 - 2kx + (k^2 - 3k + 2) = 0$$

For how many values of k does the equation have two distinct positive real roots?

A 6

B 7

C 8

D 9

E 10

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12. The interior angle of a regular polygon with n sides exceeds the interior angle of a regular polygon with m sides by exactly $18°$

How many ordered pairs of integers (n, m) with $n \geq 3$ and $m \geq 3$ satisfy this condition?

A 0

B 1

C 2

D 3

E 7

F infinitely many

13. Let c be a real constant.

Consider the equation

$$|\sin x| + |\cos x| = c$$

for $0 \le x < 2\pi$

Which of the following gives the number of distinct solutions as a function of c?

A If $0 < c < 1$, there are 8 solutions;
if $1 < c < \sqrt{2}$ there are 4 solutions;
if $c = 1$ **or** $c = \sqrt{2}$ there are 2 solutions;
otherwise there are 0 solutions

B If $0 < c < 1$, there are 0 solutions;
if $c = 1$ there are 4 solutions;
if $1 < c < \sqrt{2}$ there are 8 solutions;
if $c = \sqrt{2}$ there are 4 solutions;
otherwise there are 0 solutions

C If $0 < c < 1$, there are 0 solutions;
if $c = 1$ there are 2 solutions;
if $1 < c < \sqrt{2}$ there are 4 solutions;
if $c = \sqrt{2}$ there are 8 solutions;
otherwise there are 0 solutions

D If $0 < c < 1$, there are 4 solutions;
if $c = 1$ there are 8 solutions;
if $1 < c < \sqrt{2}$ there are 4 solutions;
if $c = \sqrt{2}$ there are 2 solutions;
otherwise there are 0 solutions

E If $0 < c < 1$, there are 0 solutions;
if $1 \le c \le \sqrt{2}$ there are 8 solutions;
otherwise there are 0 solutions

F If $0 < c < \sqrt{2}$ there are 8 solutions;
if $c = \sqrt{2}$ there are 4 solutions;
if $c > \sqrt{2}$ there are 0 solutions

14. Let a, b, c be real constants with $a > 0$ and $c \neq 0$.

Suppose the inequality $ax^2 + bx + c > 0$ has solution set

$$x < r \quad \textbf{or} \quad x > s$$

where $r < s$

In terms of r, s and c, what is the solution set of

$$a(cx)^2 + b(cx) + c > 0 \qquad ?$$

A $x < cr$ **or** $x > cs$

B $x < \frac{r}{c}$ **or** $x > \frac{s}{c}$

C $x < cs$ **or** $x > cr$

D $x < \frac{s}{c}$ **or** $x > \frac{r}{c}$

E If $c > 0$: $x < \frac{r}{c}$ **or** $x > \frac{s}{c}$
if $c < 0$: $x < \frac{s}{c}$ **or** $x > \frac{r}{c}$

F If $c > 0$: $\frac{r}{c} < x < \frac{s}{c}$
if $c < 0$: $\frac{s}{c} < x < \frac{r}{c}$

G If $c > 0$: $x < \frac{r}{c}$ **or** $x > \frac{s}{c}$
if $c < 0$: $x < \frac{r}{c}$ **or** $x > \frac{s}{c}$

15. A sequence (u_n) is defined by $u_1 > 0$ and

$$u_{n+1} = \frac{u_n}{1 + u_n}$$

for all integers $n \geq 1$

Which of the following is equal to $\sum_{n=1}^{\infty} u_n$?

A u_1

B $\frac{u_1}{1 - u_1}$

C $\frac{1}{u_1}$

D $1 + u_1$

E $\frac{1}{1 + u_1}$

F The series does not converge for any $u_1 > 0$

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16. Let

$$f(x) = x^3 - 3x + 1$$

How many distinct real solutions does the equation $f(x) = 0$ have?

A 0

B 1

C 2

D 3

E 4

F Infinitely many

17. Let a and b be positive integers and define a sequence v_n by

$$v_1 = a, \quad v_2 = b, \quad v_{n+2} = 2v_{n+1} - v_n \quad \text{for } n \geq 1$$

Suppose the highest common factor of a and b is d

Which of the following statements must be true?

A The highest common factor of v_{10} and v_{11} is 1

B v_n is divisible by d only for even n

C The highest common factor of v_n and v_{n+1} is d for all $n \geq 1$

D For all $n \geq 1$, v_n is divisible by $2d$

E If $a \neq b$ then v_n is never divisible by d

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18. How many solutions are there to the equation

$$2\sin x = |\cos x|$$

for $0° \leq x < 360°$, and what is the sum of all the solutions?

A Number of solutions $= 2$ Sum of solutions $= 180°$

B Number of solutions $= 2$ Sum of solutions $= 360°$

C Number of solutions $= 2$ Sum of solutions $= 720°$

D Number of solutions $= 4$ Sum of solutions $= 360°$

E Number of solutions $= 4$ Sum of solutions $= 540°$

F Number of solutions $= 4$ Sum of solutions $= 720°$

G Number of solutions $= 6$ Sum of solutions $= 900°$

H Number of solutions $= 8$ Sum of solutions $= 1440°$

19. A circle has centre O and radius 5

Points A and B lie on the circumference

The area of triangle AOB is 10

What is the greatest possible length of chord AB?

A $4\sqrt{5}$

B $2\sqrt{5}$

C $5\sqrt{2}$

D 8

E 6

F $7\sqrt{3}$

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20. Let $f(x) = x^3 - 3x$

Consider the following three curves:

(1) $y = f(x)$
(2) $y = f(x) + 2$
(3) the curve $y = f(x)$ reflected in the x-axis

The trapezium rule is used to estimate the signed area $\int_{-1}^{1} y\,\mathrm{d}x$ for each curve using the points $x = -1$, $x = 0$ and $x = 1$

Select the row that correctly describes whether the trapezium rule gives an overestimate, an underestimate, or an exact value for each curve

	Curve (1)	**Curve (2)**	**Curve (3)**
A	exact	exact	exact
B	exact	exact	overestimate
C	exact	underestimate	exact
D	underestimate	exact	exact
E	overestimate	exact	underestimate
F	underestimate	underestimate	overestimate